# BOUNCE BACK 2.0 <br> Whatem 

JEE MAINS \& ADVANGED ONE SHOT

# OUDN:MITO <br> (Basic $\rightarrow$ Adv) <br> (8 Marks) 

## EQUAIDIS

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$\square$ 7+ years Teaching experience
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$$
\text { Notes } \rightarrow 3 \text { OR } 4
$$

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## Quadratic Equations

Quadratic Equations

$$
\begin{array}{ll}
\rightarrow \text { deg }=2 \\
a x^{2}+b x+c=0 \quad{ }^{\alpha} \neq 0 \\
\beta, \beta=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} & \\
& =b^{2}-4 a c \\
& =\text { discriminant }
\end{array}
$$

Quadratic Equations

$$
\left.a x^{2}+b x+c=0<\begin{aligned}
& \alpha \\
& \beta
\end{aligned} \right\rvert\,
$$

(1) $\alpha+\beta=\frac{-b}{a}$
(2) $\alpha \beta=\frac{c}{a}$

$$
\begin{aligned}
& x^{2} \rightarrow \text { Coeff }=1 \\
& V_{1} x^{2}-3 x+2=0 \quad(\# \text { NVStyle }) \\
& \vee x^{2}-2 x-x+2=0 \\
& \vee x(x-2)-1(x-2)=0 \\
& \vee(x-2)(x-1)=0 \\
& \vee x=1,2 \rightarrow \text { Roods } / \text { Zeros }
\end{aligned}
$$

II Quadratic Equations
(1)

$$
\begin{aligned}
1 x^{2}-3 x+2 & =0 \\
(x-2)(x-1) & =0
\end{aligned}
$$

(2)

$$
\begin{aligned}
& 1 x^{2}-5 x+4 \\
= & (x-1)(x-4) \\
& (x-5)(x+4)
\end{aligned}
$$

(3) $1 x^{2}-x-6$

$$
(x-3)(x+2)
$$

Quadratic Equations

$$
a x^{2}+b x+c=0<\begin{aligned}
& \alpha, \alpha+\beta=\frac{-b}{a} \text {-(1) } \\
& \beta, \alpha \beta=\frac{c}{a} \text {-(2) }
\end{aligned}
$$

(1) $\alpha^{2}+\beta^{2}=(\alpha+\beta)^{2}-2 \alpha \beta$
(2) $(\alpha-\beta)^{2}=(\alpha+\beta)^{2}-4 \alpha \beta$
(3) $\alpha^{3}+\beta^{3}=(\alpha+\beta)^{3}-3 \alpha \beta(\alpha+\beta)$
(4) $\underline{\alpha}^{4}+\beta^{4}=\left(\alpha^{2}+\beta^{2}\right)^{2}-2 \alpha^{2} \beta^{2}$

Quadratic Equations

$$
\begin{array}{rlr}
(\alpha-\beta)^{2} & =(\alpha+\beta)^{2}-4 \alpha \beta \\
& =\left(\frac{(b b}{a}\right)^{2}-4\left(\frac{c}{a}\right) & *|\alpha-\beta|=\frac{\sqrt{D}}{|a|} \\
& =\frac{b^{2}}{a^{2}}-\frac{4 a c}{a^{2}} & Q E<_{-2}^{5} \\
(\alpha-\beta)^{2} & =\frac{b^{2}-4 a c}{a^{2}} \quad & x^{2}-(\text { sum }) x+\text { Product }=0 \\
|\alpha-\beta| & =\frac{\sqrt{b^{2}-4 a c}}{|a|}=\frac{\sqrt{D}}{|a|} & x^{2}-3 x-10=0
\end{array}
$$

Let $\underline{f(x)}$ be a quadratic polynomial such that $f(-2)+f(3)=0$. If
one of the roots of $f(x)=0$ is -1 ) then the sum of the roots of
\#chirdi $f(x)=0$ is equal to:
(JEE 2022)
A. $11 / 3$ (1) $f(-2)+f(3)=0$
B. $7 / 3$
C. $13 / 3$
(2) $f(x)=k(x+1)(x-\alpha)$
D. $14 / 3$
\& $K(-1)(-2-\alpha)+K(4)(3-\alpha)=0$

$$
\begin{aligned}
& \Rightarrow-1+\alpha \\
& \Rightarrow-1+\frac{14}{3} \\
& \Rightarrow \text { II }
\end{aligned}
$$

$2+\alpha+12-4 \alpha=0$
$\therefore \alpha=\frac{14}{3}$

* Sum of Roots

Let $\alpha, \beta$ be the roots of the equation
$x^{2}-\sqrt{2} x+\sqrt{6}=0$ and $\underbrace{\frac{1}{\alpha^{2}}+1, \frac{1}{\beta^{2}}+1}$ be the
roots of the equation $\sqrt{x^{2}+a x+b=0}$. Then the roots of the equation $x^{2}-(a+b-2) x+(a+b+2)$ $=0$ are :
^ non-real complex numbers
real and both negative
c. real and both positive
D. real and exactly one of them is positive

$$
\begin{aligned}
& x^{2}-\sqrt{2} x+\sqrt{6}=0<\alpha<\alpha \beta=\sqrt{6} \\
& -\frac{1}{\alpha^{2}}-\frac{1}{\beta^{2}}-2=a \\
& x^{2}+a x+b=0<\begin{array}{l}
\frac{1}{\alpha^{2}}+1 \\
\frac{1}{\beta^{2}}+1
\end{array} \\
& \frac{\left(\frac{1}{\alpha^{2}}+1\right)\left(\frac{1}{\beta^{2}}+1\right)=b}{\frac{1}{(\alpha \beta)^{2}}+1-2=a+b} \\
& x^{2}-\left(\frac{-5}{6}-2\right) x+\left(\frac{-5}{6}+2\right)=0<\text { ? } \\
& 6 x^{2}+17 x+7=0 \quad 7 \times 3 \times 2 \\
& \frac{1}{6}-1=a+b \\
& 6 x^{2}+14 x+3 x+7=0 \\
& a+b_{0}=\frac{-5}{6} \\
& (2 x+1)(3 x+7)=0 \\
& x=\frac{-1}{2},-\frac{7}{3}
\end{aligned}
$$

The minimum value of the sum of the squares of the roots of $x^{2}+(3-a) x+1=2 a$ is :
A. 4

$$
x^{2}+(3-a) x+(1-2 a)=0<\beta
$$

(JEE n2022)
B. 5
C. 6
D. 8

$$
\left(\alpha^{2}+\beta^{2}\right)_{\text {mini }}=(\alpha+\beta)^{2}-2 \alpha \beta
$$

$$
\begin{aligned}
& \left(\alpha^{2}\right. \\
& d i)
\end{aligned}
$$

$\therefore a=1$
$(c$ for chindi)

$$
\begin{aligned}
& =(a-3)^{2}-2(1-2 a) \\
& =a^{2}-6 a+9-2+4 a \\
& =a^{2}-2 a+1+6 \\
& =(a-x)^{2}+6
\end{aligned}
$$

If the sum of the squares of the reciprocals of the roots $\alpha$ and $\beta$
of the equation $3 x^{2}+\lambda x-1=0$ is 15 , then $6\left(\alpha^{3}+\beta^{3}\right)^{2}$ is equal to:
A. 18
(JEEM 2022)
B. 24
C. 36

$$
3 x^{2}+\lambda x-1=0<\begin{aligned}
& \alpha \\
& \beta
\end{aligned} \alpha^{3}+\beta^{3}=?
$$

$$
\alpha+\beta=\frac{-\lambda}{3}=( \pm 1)
$$

$$
\alpha \beta=\frac{-1}{3}
$$

$$
\begin{aligned}
& \frac{1}{\alpha^{2}}+\frac{1}{\beta^{2}}=15 \\
& \frac{(\alpha+\beta)^{2}-2 \alpha \beta}{(\alpha \beta)^{2}}=15 \quad\left\{\begin{array}{l}
\frac{\lambda^{2}}{x}+\frac{6}{\alpha}=\frac{15}{\alpha} \\
\frac{\lambda^{2}}{9}-2\left(\frac{-1}{3}\right) \\
\lambda^{2}=9 \\
\lambda= \pm 3
\end{array}\right.
\end{aligned}
$$

$$
\begin{align*}
\underbrace{\alpha^{3}+\beta^{3}} & =(\alpha+\beta)^{3}-3 \alpha \beta(\alpha+\beta) \\
& =( \pm 1)^{3}-\beta\left(\frac{-1}{\beta}\right)( \pm 1) \\
& =( \pm 1)+( \pm 1) \quad 6\left(\alpha^{3}+\beta^{3}\right)^{2} \\
& =2,0,-2=6( \pm 2)^{2}
\end{align*}
$$

Suppose $a, b$ denote the distinct real roots of the quadratic polynomial $x^{2}+20 x-2020$ and suppose $c, d$ denote the distinct complex roots of the quadratic polynomial $x^{2}-20 x+2020$. Then the value of $a c(a-c)+a d(a-d)+b c(b-c)+b d(b-d)$ is
A. 0
[Adv. 2020]
B. 8000
C. 8080

$$
\begin{aligned}
& a+b=-20 \\
& a b=-2020
\end{aligned}
$$

LD. 16000

$$
x^{2}+20 x-2020=0 \mathcal{l}_{b}^{a}
$$

$$
\begin{aligned}
\underline{c+d} & =20 \\
c d & =2020
\end{aligned}
$$

$$
\begin{aligned}
R_{\text {eq }} & \Rightarrow a^{2} c-a c^{2}+a^{2} d-a d^{2}+\underbrace{b^{2} c}-b c^{2}+b^{2} d-b d^{2} \\
& \Rightarrow a^{2}(c+d)+b^{2}(c+d)-c^{2}(a+b)-d^{2}(a+b) \\
& \Rightarrow a^{2}(20)+b^{2}(20)+c^{2}(20)+d^{2}(20) \\
& \Rightarrow 20\left(a^{2}+b^{2}+c^{2}+d^{2}\right) \\
& \Rightarrow 20((a+b)^{2}-2 a b+(\underbrace{c+d)^{2}}-2 c d) \\
& \Rightarrow 20(400+2(2020)+400-2(2020)) \\
& \Rightarrow 16000
\end{aligned}
$$



## \# Newton's Method

Newton's Method: Powers of Roots

Let $\alpha$ and $\beta$, are the roots of the quadratic equation $a x^{2}+b x+c=0$, and $S_{n}=\alpha^{n} \pm \beta^{n}$ then $a S_{n}+b S_{n-1}+c S_{n-2}=$ 0

$$
\begin{aligned}
& a\left[x^{2}+b\left(x+c(1)=0 \alpha_{\beta}^{\alpha} S_{n}=\alpha^{n} \pm \beta^{n}\right.\right. \\
& a S_{n}+b S_{n-1}+c S_{n-2}=0 \quad \begin{array}{l}
x^{2} \rightarrow S_{n} \\
x \rightarrow S_{n-1} \\
1 \rightarrow S_{n-2}
\end{array}
\end{aligned}
$$

Let $\alpha$ and $\beta$ be the roots of $x^{2}-6 x-2=0$, with $\alpha>\beta$. If
$a_{n}=\alpha^{n}-\beta^{n}$ for $n \geq 1$, then the value of

A. 1

$$
\begin{aligned}
& x^{2}-6 \theta-2=0<\beta \\
& * a_{n}=\alpha^{n}-\beta^{n} \text { \#Newton }
\end{aligned}
$$

JEE Adv. 2011 \& JEE Main 2015
B. 2 (JEE Main 2021)
e. 3
D. 4

$$
\begin{gathered}
a_{n}-6 a_{n-1}-2 a_{n-2}=0 \\
n=10 \\
\frac{a_{10}-6 a_{9}-2 a_{8}}{}=0 \\
\frac{a_{10}-2 a_{8}}{2 a_{9}}=\frac{6}{2}=3
\end{gathered}
$$

Let $\alpha$ and $\beta$ be the roots of the equation, $5 x^{2}+6 x-2=0$. If $S_{n}=\alpha^{n}+\beta^{n}, n=1,2,3, \ldots .$, then :
A. $6 \mathrm{~S}_{6}+5 \mathrm{~S}_{5}=2 \mathrm{~S}_{4}$
B. $6 \mathrm{~S}_{6}+5 \mathrm{~S}_{5}+2 \mathrm{~S}_{4}=0$

$$
5 S_{n}+6 S_{n-1}-2 S_{n-2}=0
$$

c. $5 S_{6}+6 S_{5}=2 S_{4}$
D. $5 \mathrm{~S}_{6}+6 \mathrm{~S}_{5}+2 \mathrm{~S}_{4}=0$

$$
n=6
$$

$$
S S_{6}+6 S_{5}-2 S_{4}=0
$$

For a natural number $n$, let $a_{n}=19^{n}-12 n$. Then,
the value of $\frac{51 \alpha_{9}-\alpha_{10}}{57 \alpha_{8}}$ is

$$
a_{n}=\alpha^{n}-\beta^{n} \quad \text { (TEE } m \text { 2022) }
$$

$$
\begin{aligned}
& \left.\begin{array}{l}
x^{2}-(S) x+P=0 \\
x^{2}-31 x+228=0<12
\end{array} \quad \begin{array}{l}
\alpha=19 \\
\beta=12
\end{array}\right\} \text { Roots } \\
& \begin{array}{r}
\overrightarrow{a_{10}}-31 a_{9}+228 a_{8}=0
\end{array} \\
& (4)=\frac{228 a_{8}}{57 q_{8}}=\frac{31 a_{9}-a_{10}}{57 a_{8}}
\end{aligned}
$$

Let $\alpha, \beta(\alpha>\beta)$ be the roots of the quadratic equation $x^{2}-x-4=0$. If $P_{n}=\alpha^{n}-\beta^{n}, n \in \mathbb{N}$, then $\Rightarrow \frac{P_{15} P_{16}^{\prime}-P_{14} P_{16}^{\prime}-P_{15}^{2}+P_{14} P_{15}}{P_{13} \mathrm{P}_{14}}$ is equal to $\qquad$ (2022)

$$
\begin{aligned}
& P_{13} P_{14} \\
& \Rightarrow \frac{P_{16}\left(P_{15}-P_{14}\right)-P_{15}\left(P_{15}-P_{14}\right)}{P_{13} P_{14}}\left(\begin{array}{l}
P_{n}-P_{n-1}-4 P_{n-2}=0 \\
P_{15}-P_{14}
\end{array}=4 P_{13}\right. \\
& P_{16}-P_{15}=4 P_{14} \\
& \Rightarrow \frac{\left(P_{15}-P_{14}\right)\left(P_{16}-P_{15}\right)}{P_{14}} \Rightarrow \frac{(4 P / 3)(4 P / 14)}{9 / 3 / 4} \Rightarrow 16
\end{aligned}
$$



Identity

Identity :

Let $a x^{2}+b x+c=0$ be $a$ quadratic equation. Now, if this quadratic equation has more than two distinct roots then it becomes an identity and in this case $\mathbf{a}=\mathbf{b}=\mathbf{c}=\mathbf{0}$.

* $(x+1)^{2}=x^{2}+2 x+1$ $\infty$ Root

$$
a x^{2}+b x+c=0
$$

identity if $a=b=c=0$

$$
0 x^{2}+0 x+0=0 \quad \infty R_{0 o t}
$$

For what values of $p$, the equation $(p+2)(p-1) \underline{2}^{2}+(p-1)(2 p+1) \underline{x}+p^{2}-1=0$ has more than two roots.
$Q E=0$ - more than 2 Root

$$
\begin{array}{r}
a=b=c=0 \\
(p+2)(p-1)=0 \\
(p-1)(2 p+1)=0 \\
p^{2}-1=0
\end{array}
$$



## Nature of Roots

## Nature of Roots :

i. If $\mathrm{D}>\mathrm{O} \Rightarrow$ roots are real and distinct.
ii. If $D=0 \Rightarrow$ roots are equal.
iii. If $\underline{\underline{\mathrm{D}<0}} \Rightarrow$ roots are imaginary.

1. If coefficients of the quadratic equation arerational then its irrational roots always occur in pair. If $p+\sqrt{q}$ is one of the roots then other root will be $p-\sqrt{q}$
2. If coefficients of the quadratic equation arereal then its imaginary roots always occur in complex conjugate pair. If $p+i q$ is one of the roots then other root will be $p-i q$
3. If coefficients of the quadratic equation arereal and $D=$ perfect square then the roots are rational

Nature of Roots :

$$
\begin{aligned}
& a x^{2}+\underline{b} x+c=0 \quad \frac{-b \pm \sqrt{25}}{2 a}=\text { Roots }=\frac{-b \pm s}{2 a} \\
& \text { * } \overline{a, b, c \rightarrow \text { Ration }} \text { Irrational } \rightarrow \text { pair } p+\sqrt{q}, p-\sqrt{q} \\
& \text { * } a, b, c \rightarrow \text { Real } \quad \text { Imam } \rightarrow \text { pair } p+i q, p-i q \\
& \begin{array}{l}
\text { * } D=p . s q . \\
a, b, c \in \text { Rational }
\end{array} \quad \text { Roots }=\text { Rational }
\end{aligned}
$$

The quadratic equation with rational coefficients whose one root is $2+\sqrt{3}$ is
A. $x^{2}-4 x+1=0 \quad \alpha=(2+\sqrt{3})$
B. $x^{2}+4 x+1=0 \quad \beta=(2-\sqrt{3})$
C. $x^{2}+4 x-1=0 \quad$ Sum $=4$
D. $x^{2}+2 x+1=0 \quad$ Product $=1$

$$
x^{2}-4 x+1=0
$$

The roots of the quadratic equation $x^{2}-2(a+b) x+2\left(a^{2}+b^{2}\right)=0$ are
A. Rational and different
B. Rational and equation equal
C. Irrational and different

Concept. Imaginary and different
Rational Root $D=p . s q$
Equal $D=0$ heal
Imam. $D<0$

$$
\begin{aligned}
D & =4(a+b)^{2}-4(2)\left(a^{2}+b^{2}\right) \\
& =4\left\{a^{2}+b^{2}+2 a b-2 a^{2}-2 b^{2}\right\} \\
& =4\left\{-a^{2}-b^{2}+2 a b\right\} \\
& =-4\left\{a^{2}+b^{2}-2 a b\right\} \\
D & =-4(a-b)^{2} \\
D & <0
\end{aligned}
$$

The number of all possible positive integral values of $\alpha$ for which the roots of the quadratic equation, $6 x^{2}-11 x+\alpha=0$ are rational numbers is :
A. 3

$$
D=p . s q \Rightarrow \text { Rational }
$$

JEE M 2019
B. $2 \quad D=|2|-4(6)(\alpha)$
C. 4
D. 5

$$
\left.\begin{array}{rl}
\therefore D & =|2|-24 \alpha \\
\text { P.sq. } & =12 \mid-24=97 X \\
& =121-24(2)=73 X \\
& =121-24(3)=49 \mathrm{~V} \\
& =12 \mid-24(4)=25 \checkmark \\
& =121-24(\delta)=1 \checkmark \\
& =121-24(6)=\Theta \\
(7) & =\Theta
\end{array}\right\}
$$

$$
\alpha=3,4,5
$$



# * Graph of Quadratic 

Quadratic Expression and its Graph :
Quad. Exp $\Rightarrow$ parabola

II. Quadratic Expression and its Graph:
(1) $a>0 \cup a<0 \bigcap y=a x^{2}+b / x+c$ $x=0 \quad y=c$
(2) $D>02$ Real

$D=0$ Real $\ell_{1}$ Equal
$D<0$ No Real

(3)


$$
\text { Vertex } \equiv\left(\frac{-b}{2 a}, \frac{-D}{4 a}\right)
$$

$$
c_{2} \theta
$$



Quadratic Expression and its Graph :



Quadratic Expression and its Graph :


Draw the graph of $\mathbf{y}=\mathrm{x}^{2}-\mathbf{7 x + 1 2}$
Axis of Sym.
(1-1 $a>0$ upward $=\frac{1}{4}$

$$
\begin{gathered}
\text { s-2 } D=49-4(12)=1 \\
D>0 \quad 2 \operatorname{Rost} \\
x^{2}-7 x+12=0 \\
(x-3)(x-4)=0 \\
x=3,4
\end{gathered}
$$


up: Range $\Rightarrow\left[\frac{-D}{4 a}, \infty\right)$
$y_{v}=$ ?
down: Range $\Rightarrow\left(-\infty, \frac{-D}{4 a}\right]$

$$
\begin{gathered}
y=x^{2}-7 x+12 \\
\therefore \frac{d y}{d x}=2 x-7=0 \\
x_{v}=7 / 2
\end{gathered}
$$



Draw the graph of $\mathrm{y}=-\mathrm{x}^{2}+\mathrm{x}-\mathbf{1}$
Range: $\left(-\infty, \frac{-3}{4}\right]$
d-1 $a<0 \bigcap$
赼 $D=1^{2}-4(-1)(-1)$
$D=-3$
$D<0$ No Real Root


1-3 $(0,-1)$ x-axis Kocut nu
s-4 $V \equiv\left(\frac{-b}{2 a}, \frac{-D}{4 a}\right) \equiv\left(\frac{-1}{2(-1)}, \frac{3}{4(-1)}\right) \equiv\left(\frac{1}{2}, \frac{-3}{4}\right)$

Sign of Quadratic Expression:

* If $a>0$ and $D<0$ then $a x^{2}+b x+c$ is always positive
* If $a<0$ and $D<0$ then $a x^{2}+b x+c$ is always negative


$$
\begin{aligned}
& y=x^{2}+x+1 \\
& \therefore a>0 \\
& D<0
\end{aligned} \begin{aligned}
& x^{2}+x+1 \Rightarrow \text { Always } \\
& (-10)^{2}+(-10)+1 \Rightarrow \theta
\end{aligned}
$$

If the numbers appeared on the two throws of a fair six faced die are $\alpha$ and $\beta$, then the probability that $x^{2}+\alpha x+\beta>0$, for all $x \in R$, is :
(JEE 2022)
A. $17 / 36$ $\square$
$\square$
B. $4 / 9$

$$
\alpha, \beta \in\{1,2,3,4,5,6\}
$$

C. $1 / 2$

$$
x^{2}+\alpha x+\beta \Rightarrow \text { Always Positive }
$$

$a>0$
D. $19 / 36$

$$
D=\alpha^{2}-4 \beta<0
$$

$$
\begin{aligned}
P & =\frac{\text { far. }}{\text { Total }} \\
& =\frac{17}{6 \times 6}
\end{aligned}
$$

$\alpha^{2}<24$


Consider the graph of quadratic trinomial $y=a x^{2}+b x+c a s$ shown below where $x_{1}$ and $x_{2}$ are roots of the equation $a x^{2}+b x+c=0$. Which of the following is/are correct?

MCQ
A. $a-b-c<0$
\% $b c<0$
c. $b>0$
D. $b$ and $c$ have the same sian different from $a$

$$
\frac{+b}{2 \phi}=t
$$



If $a x^{2}+b x+c=0$ has imaginary roots and $a, b, c>0$. Then possible graph of $y=a x^{2}+b x+c$ is :



$$
\begin{aligned}
& \quad a>0 \\
& b>0 \\
& c>0 \\
& \frac{-b}{2 a}=\frac{-\Theta}{2 \oplus}=\Theta
\end{aligned}
$$

C.




## Symmetric Expression :

Expressions in $\alpha$ and $\beta$, which do not change by interchanging $\alpha$ and $\beta$.

Some examples of symmetric expressions are
i. $\underbrace{\alpha^{2}+\beta^{2}}$
ii. $\underbrace{\alpha^{2}+\alpha \beta+\beta^{2}}$
iif. $\underbrace{\frac{1}{\alpha}+\frac{1}{\beta}}$
iv. $\underbrace{\frac{\alpha}{\beta}+\frac{\beta}{\alpha}}$
V. $\underbrace{\alpha^{2} \beta+\beta^{2} \alpha}$
vi. $\underbrace{\left(\frac{\alpha}{\beta}\right)^{2}+\left(\frac{\beta}{\alpha}\right)^{2}}$
vii. $\underbrace{\alpha^{3}+\beta^{3}}$
viii. $\underbrace{\alpha^{4}+\beta^{4}}$

$$
\begin{aligned}
& \begin{array}{l}
\text { Q) } a x^{2}+b x+c=0<_{\beta}^{\alpha} \\
\underbrace{\underbrace{(x-1)}}{ }^{\text {New Q.E }=0}<\frac{\alpha+1}{\beta+1}
\end{array} \\
& \underline{\underline{M-1}} S=(\alpha+1)+(\beta+1) \\
& p=(\alpha+1)(\beta+1) \\
& x^{2}-5 x+P=0 \\
& \alpha+1=x \\
& \therefore \alpha=(x-1) \\
& a(x-1)^{2}+b(x-1)+c=0 \\
& P=(\alpha-3)(\beta-3) \quad \delta=\alpha-3+\beta-3 \\
& =\alpha \beta-3(\alpha+\beta)+9=\alpha+\beta-6 \\
& =\frac{7}{2}-3\left(-\frac{5}{2}\right)+9=-\frac{5}{2}-6=-17 / 2 \quad \begin{array}{l}
\therefore=x+3 \\
x^{2}+\frac{17}{2} x+20=0 \quad 2 x^{2}+17 x+40=0
\end{array} \\
& =20
\end{aligned}
$$

$a x^{2}+b x+c=0<_{\beta}^{\alpha} \quad$ New $Q E<$ ?

$$
\begin{aligned}
& -\alpha=x \\
& \therefore \alpha=-x
\end{aligned}
$$

$\left\{\begin{array}{l|ll}\text { New Roots } & \text { Replace } & \begin{array}{c}\alpha+k=x \\ -\alpha,-\beta \\ \alpha+k, \beta+k \\ \end{array} x \rightarrow(-x) \\ x \rightarrow(x-k) & \therefore \alpha=x-k \\ \alpha-k, \beta-k & x \rightarrow(x+k) & \frac{\alpha}{k}=x \\ k \alpha, k \beta & x \rightarrow \frac{x}{k} & \alpha=k x \\ \frac{\alpha}{k}, \frac{B}{k} & x \rightarrow k x & \alpha^{n}=x \\ \alpha^{n}, \beta^{n} & x \rightarrow x^{1 / n} & \alpha=x^{1 / n} \\ \alpha+2, \beta+3 \lambda & & \end{array}\right.$

If $\alpha$ and $\beta$ are roots of $2 x^{2}-7 x+6=0$, then the quadratic equation whose roots are $-2 / \alpha,-2 / \beta$ is
A. $3 x^{2}+7 x+4=0$
B. $3 x^{2}-7 x+4=0$
C. $6 x^{2}+7 x+2=0$
D. $6 x^{2}-7 x+2=0$

New $Q E=0<\begin{aligned} & -2 / k \\ & -2 / \beta\end{aligned}$

$$
\begin{aligned}
& \frac{-2}{\alpha}=x \\
& \alpha=\frac{-2}{x} \\
& x \rightarrow \frac{-2}{x}
\end{aligned}
$$

$$
\begin{gathered}
2\left(\frac{-2}{x}\right)^{2}-7\left(\frac{-2}{x}\right)+6=0 \\
x^{2}\left(\frac{8}{x^{2}}+\frac{14}{x}+6=0\right. \\
\frac{1}{2}+14 x+6 x^{2}=0 \\
3 x^{2}+7 x+4=0
\end{gathered}
$$

Bi Quadratic Equation

$$
\begin{aligned}
& a x^{4}+b x^{3}+c x^{2}+b x+a=0 \\
& \text { * Sym } \\
& \frac{\text { divide by } x^{2}}{\sqrt{2}} \text { :- } \\
& x+\frac{1}{x}=t \\
& a x^{2}+b x+c+\frac{b}{x}+\frac{a}{x^{2}}=0 \\
& a\left(x^{2}+\frac{1}{x^{2}}\right)+b\left(x+\frac{1}{x}\right)+c=0 \\
& a\left(\left(x+\frac{1}{x}\right)^{2}-2\right)+b\left(x+\frac{1}{x}\right)+c=0
\end{aligned}
$$

The sum of the cube of all the roots of the equation $x^{4}-3 x^{3}-2 x^{2}+3 x+1=0$ is_.

$$
\begin{aligned}
& x^{4}-3 x^{3}-2 x^{2}+3 x+1=0<_{8}^{\alpha} \\
& x^{2}-3 x-2+\frac{3}{x}+\frac{1}{x^{2}}=0 \\
& \left(x-\frac{1}{x}\right)^{2}+\not 2-3\left(x-\frac{1}{x}\right)-2=0 \\
& t^{2}-3 t=0 \\
& t(t-3)=0 \\
& t=0 \text { OR } 3
\end{aligned}
$$

Req

$$
\begin{aligned}
& \Rightarrow \alpha^{3}+\beta^{3}+\gamma^{3}+\delta^{3} \\
& \Rightarrow 1^{\prime}+\left(-\gamma^{3}+(\gamma+\delta)^{3}-3 \gamma \delta(\gamma+\delta)\right.
\end{aligned}
$$

$$
\Rightarrow 0+27+3(3)
$$

$\alpha, \beta$
$\left.\begin{array}{l}x-\frac{1}{x}=0 \\ x^{2}=1 \\ x= \pm 1 \\ \alpha=1 \\ \beta=-1\end{array}\right\}$

$$
\begin{aligned}
& \gamma, 8 \\
& x-\frac{1}{x}=3 \\
& x^{2}-3 x-1=0<_{8}^{\gamma} \\
& \frac{\gamma+8=3}{\gamma \delta=-1}
\end{aligned}
$$

The number of real solutions of the equation $e^{4 x}+4 e^{3 x}-58 e^{2 x}+4 e^{x}+1=0$ is_
(2022)

Let, $e^{x}=t$
$t^{4}+4 t^{3}-58 t^{2}+4 t+1=0$

$$
\begin{aligned}
& t^{2}+4 t-58+\frac{\sqrt[5]{t}}{t}+\frac{1}{t^{2}}=0 \\
& \left(t+\frac{1}{t}\right)^{2}-2+4\left(t+\frac{1}{t}\right)-58=0
\end{aligned}
$$

$\underbrace{t^{2}+\frac{1}{t^{2}}=\left(t+\frac{1}{t}\right)^{2}-2}$

$$
\begin{aligned}
& z^{2}+4 z-60=0 \\
& (z+10)(z-6)=0 \\
& z=-10 / 6
\end{aligned}
$$

The sum of all the real roots of the equation
$\left(e^{2 x}-4\right)(\underbrace{2 x}-5 e^{x}+1)=0$ is
(2022)

Pirn
A. $\log _{e} 3$
(\#chindi)
B. $-\log _{e} 3$
$\left(e^{x}-2\right)\left(e^{x}+2\right)\left(3 e^{x}-1\right)\left(2 e^{x}-1\right)=0$
C. $\log _{e} 6$
D. $-\log _{e} 6$

$$
e^{x}=2, \not-6, \frac{1}{3}, \frac{1}{2}
$$

$$
\begin{gathered}
e^{x}=t \\
\left(t^{2}-4\right)\left(6 t^{2}-5 t+1\right) \\
(t-2)(t+2)(3 t-1)(2 t-1) \\
e^{x}=2 \\
x=\ln 2
\end{gathered}
$$

$$
\therefore x=\ln 2, \ln \frac{1}{3}, \ln \frac{1}{2}
$$

$$
\begin{aligned}
\sum x & =\ln 2+\ln \frac{1}{3}+\ln \frac{1}{2} \\
& =\ln 3^{-1} \\
& =-\ln 3
\end{aligned}
$$

$Q E 1 \ll_{\beta}^{\alpha}$ $Q E 2<\gamma$

Condition of Common Roots:

Condition for both the common roots

$$
\begin{array}{ll}
a_{1} x^{2}+b_{1} x+c_{1}=0<_{\beta}^{\alpha} & 1 x^{2}-3 x+2=0< \\
a_{2} x^{2}+b_{2} x+c_{2}=0 \ll \\
\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}} & 2 x^{2} \\
\text { then both Roods are Common. } & \frac{1}{2}=\frac{-3}{-6}=\frac{2}{4}=k
\end{array}
$$

III Condition of Common Roots:

* Condition for one common roots :

$$
\begin{aligned}
& \left.a_{1} x^{2}+b_{1} x+c_{1}=0 \lll \gamma\right) \\
& a_{2} x^{2}+b_{2} x+c_{2}=0 \\
& \left\langle\begin{array}{l}
\alpha=\sqrt{\beta} \\
\gamma
\end{array}\right. \\
& \otimes \beta=\frac{c_{1}}{a_{1}} \\
& \text { Method:- } a_{2}\left(a_{1} \alpha{ }^{2}+b_{1} Q+c_{1}=0\right) \\
& \beta=\frac{c_{1}}{a_{1} \Omega} \\
& \stackrel{S-1}{=} \\
& \stackrel{1-2}{ } \\
& \frac{\sqrt{a_{1}} a_{2} \alpha^{2}+b_{2}\left(\alpha+c_{2}=0\right)}{\alpha=-} \\
& (\alpha)(\gamma)=v
\end{aligned}
$$

## Note :

## \# chalaki

* Given one root is common but one of the QE has D<0 then both roots will be common

$$
\begin{aligned}
& \gamma \underline{a_{1} x^{2}+b_{1} x+c_{1}=0}<\begin{array}{l}
\alpha=p+i q \\
\beta=p-i q
\end{array} \\
& \checkmark_{\#} \underline{a_{2} x^{2}+b_{2} x+c_{2}=0}<\begin{array}{l}
\alpha=p+i q \\
\gamma=p-i q
\end{array} \\
& \left(\frac{D<0}{\text { image. }} \Rightarrow \text { pair } \frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}}\right.
\end{aligned}
$$

Let $\lambda \neq 0$ be in R. If $\alpha$ and $\beta$ are roots of the equation, $x^{2}-x+2 \lambda=0$ and $\alpha$ and $\gamma$ are the roots of the equation,
$3 x^{2}-10 x+27 \lambda=0$, the $\beta \gamma / \lambda$ is equal to :
A. 27

$$
x^{2}-x+2 \lambda=0
$$

$$
\alpha=3
$$

B. 18
C. 9


SEE MAIN 2020
D. 36


$$
\begin{array}{l|l}
(\alpha)(\beta)=2 \lambda & (\alpha)(\gamma)=9 \lambda \\
(3 \lambda)(\beta)=2 \gamma & (3 \lambda)(\gamma)=9 \lambda \\
\therefore \beta=\frac{2}{3} & \therefore \gamma=3
\end{array}
$$

$$
\begin{aligned}
& (3 \lambda)^{2}-3 \lambda+2 \lambda=0 \\
& 9 \lambda^{2}=\lambda \\
& \lambda=\frac{1}{9}
\end{aligned}
$$

Let $a, b \in \mathrm{R}, a \neq 0$ be such that the equation, $a x^{2}-2 b x+5=0$ has a repeated root $\alpha$, which is also a root of the equation, $x^{2}-2 b x-10=0$. If $\beta$ is the other root of this equation, then $\alpha^{2}+\beta^{2}$ is equal to :
(a) 25
(b) 26
(c) 28
(d) 24 (PY.Q)

$$
\begin{aligned}
& a x^{2}-2 b x+s=0<_{\alpha}^{\alpha} \\
& \frac{x^{2}-2 b x-10=0}{\alpha} \\
& \frac{b^{2}}{a^{2}}-26\left(\frac{b}{a}\right)-10=0 \\
& \frac{5 a}{a^{2}}-\frac{2(5 x x)}{a}-10=0 \\
& \text { tEE MAIN } 2020 \\
& \alpha^{2}+\beta^{2} \\
& \Rightarrow(\alpha+\beta)^{2}-2 \alpha \beta \\
& \Rightarrow 4 b^{2}+20 \\
& \Rightarrow 4\left(\frac{5}{4}\right)+20 \\
& \Rightarrow 25
\end{aligned}
$$

Let $\mathrm{a}, \mathrm{b} \in \mathrm{R}$ be such that the equation $\mathrm{ax}^{2}-2 \mathrm{bx}+15=0$ has a repeated root $\alpha$. If $\alpha$ and $\beta$ are the roots of the equation $x^{2}-2 b x+21=0$, then $\alpha^{2}+\beta^{2}$ is equal to :
A. 37
(2022)
B. 58
C. 68
D. 92

If $a, b, c \in R$ and equations $\breve{a}^{\prime} x^{2}+b x+c=0$ and $x^{2}+2 x+9=0$ have a common root, then find $\mathrm{a}: \mathrm{b}: \mathrm{c}$.
lek

$$
D=4-4(9)
$$

$D<0$
\# Chalaki
both

## (

Location of the Roots:

$$
x_{0}=3
$$

1. Both the roots are greater than $x_{0}$;

Both $\alpha, \beta>3$

(2) $\frac{-b}{2 a}>3$
(3) $a f(3)>0$

## Location of the Roots:

## 2. Both the roots are less than $x_{0}$ :



Conditions:-

i) $D \geqslant 0$
ii) $\frac{-b}{2 q}<x_{0}$
iii) $a f\left(x_{0}\right)>0$

If both the roots of quadratic equation $x^{2}-2 k x+k^{2}+k-5=0$ are
(up)
A. $(5,6]$
c. $(-\infty, 4)$
B. $(6, \infty)$
D. $[4,5]$

SEE MAIN 2005
(1) $D \geqslant 0$
(2) $\frac{-b}{2 a}<5$
(3) $1 . f(s)>0$
(1)

Location of the Roots:
3. One root less than $x_{0}$ and other greater than $x_{0}$ :



$a=\oplus$

Find the value of k for which one root of the equation of $\mathbf{x}^{2}-(\mathrm{k}+1) \mathrm{x}+\mathrm{k}^{2}+\mathrm{k}-8=0$ exceed 2 and other is smaller than 2.


$$
\begin{aligned}
& a \cdot f(2)<0 \\
& 1\left\{4-(k+1)(2)+k^{2}+k-8\right\}<0 \\
& 4-2 k-2+k^{2}+k-8<0 \\
& k^{2}-k-6<0 \\
& (k-3)(k+2)<0
\end{aligned}
$$

## Location of the Roots:

4. Both root between $x_{1}$ and $x_{2}: f(x)=a x^{2}+b x+c$
$\left.\left.\begin{array}{ll}a=+ \\ \left\{\left(x_{1}\right)=+\right.\end{array}\right\} \quad \begin{array}{l}a=0 \\ \left\{\begin{array}{l}a \\ f\left(x_{2}\right)=+0\end{array}\right. \\ f\left(x_{2}\right)=0\end{array}\right\}$
( $10 \geqslant 0$
(2) $x_{1}<\frac{-b}{2 a}<x_{2}$
(3) $a f\left(x_{1}\right)>0$
(3) a. $f\left(x_{2}\right)>0$

If both the roots of quadratic equation $x^{2}-m x+4=0$ are real and distinct and they lie in the interval [1,5], then $m$ lies in the interval
A. $(-5,-4)$

ป.) $(4,5)$
C. $(5,6)$
D. $(3,4)$


$$
1 x^{2}-m x+4=0
$$

(up)

Conditions:-
(1) $D>0$
(2) $f(1)>0$
(3) $f(5)>0$
(4) $1<\frac{-b}{2 a}<5$
(1) $D>0$

$$
\begin{aligned}
& m^{2}-16>0 \\
& (m-4)(m+4)>0 \\
& \frac{+4}{+0}-0+
\end{aligned}
$$

$$
\text { (2) } \quad \begin{aligned}
& 1-m+4>0 \\
& m<5
\end{aligned}
$$

(3)
$2 s-5 m+4>0$ $m<\frac{29}{5}$
(4)

$$
\begin{aligned}
& 1<\frac{m}{2}<5 \\
& 2<m<10
\end{aligned}
$$

Location of the Roots:
5. Exactly one root between $x_{1}$ and $x_{2}$ :
\#NVTie


(1) $f\left(x_{1}\right) \cdot f\left(x_{2}\right) \leqslant 0 \Rightarrow$ Solve End points $\Rightarrow$ Verify

Location of the Roots:
6. Both root outside $x_{1}$ and $x_{2}$ :

$$
\begin{aligned}
& a=- \\
& f\left(n_{1}\right)=+
\end{aligned}
$$


(1) $a f\left(x_{1}\right)<0$
(2) $a f\left(x_{2}\right)<0$

The set of all real values of $\lambda$ for which the quadratic equations, $\left(\lambda^{2}+1\right) x^{2}-4 \lambda x+2=0$ always have exactly one root in the interval ( $\underline{\underline{0,1})}$ is :
Ques
A. $(0,2)$
B. $(2,4)$
C. $(1,3]$
D. $(-3,-1)$

$$
a=\lambda^{2}+1=\oplus(u p)
$$

(1)


$$
\begin{aligned}
& \text { f(0)} \cdot f(1) \leqslant 0 \\
& 2\left\{\lambda^{2}+1-4 \lambda+2\right\} \leqslant 0 \\
& \lambda^{2}-4 \lambda+3 \leqslant 0 \\
& (\lambda-1)(\lambda-3) \leqslant 0 \\
& \lambda \in(1,3]
\end{aligned}
$$

$$
\begin{gathered}
C=1=1 \\
2 x^{2}-4 x+2=0 \\
x^{2}-2 x+1=0 \\
(x-1)^{2}=0
\end{gathered} \quad \begin{aligned}
& 10 x^{2}-12 x+2=0 \\
& 5 x^{2}-6 x+1=0 \\
& 5 x^{2}-5 x-x+1=0 \\
& (5 x-1)(x-1)=0 \\
& \left.x=1, \frac{1}{5}\right)
\end{aligned}
$$



Theory of Equations:
For Quadratic Equation : change - not change - change - net change...

$$
a x^{2}+b x+c=0 \quad(d y=2)
$$

$\underline{\underline{1-1}}$

$$
\begin{align*}
& x^{2}+\frac{b}{a} \cdot x^{\top} \frac{c}{a}=0 \ll \\
& \left.\alpha+\beta=\frac{-b}{a}\right) \text {-(1) } \\
& \alpha \beta=\frac{c}{a} \tag{2}
\end{align*}
$$

III Theory of Equations:

$$
a x^{3}+b x^{2}+c x+d=0
$$

1-1

$$
\begin{gathered}
x^{3}+\frac{b}{a} x^{2}+\frac{c}{a} x+\frac{d}{a}=0<\beta \\
\alpha+\beta+\gamma=\frac{-b}{a} \\
\alpha \beta+\beta \gamma+\gamma \alpha=\frac{c}{a} \\
\alpha \beta \gamma=-\frac{d}{a}
\end{gathered}
$$

Theory of Equations:

For Bi-quadratic Equation :

$$
\begin{aligned}
& x^{4}+\frac{-}{a} x^{3}+\frac{c}{a} x^{2}+\frac{d}{a} x+\frac{e}{a}=0 \\
& \alpha+\beta+\gamma+\delta=\frac{-b}{a} \\
& \alpha \beta+\beta \gamma+\gamma \delta+\delta \alpha+\alpha \gamma+\delta \beta=\frac{c}{a} \\
& \alpha \beta \gamma+\beta \gamma \delta+\alpha \gamma \delta+\alpha \beta \delta=-\frac{d}{a} \\
& \alpha \beta \gamma \delta=\frac{e}{a}
\end{aligned}
$$

Solve the cubic $4 x^{3}+16 x^{2}-9 x-36=0$, the sum of its two roots being equal to zero.

$$
\begin{aligned}
& 4 x^{3}+16 x^{2}-9 x-36=0 \\
& x^{3}+4 x^{2}-\frac{9}{4} x-9=0 \\
& \alpha+(-\not q)+\beta=-4 \\
& (\alpha)(-\alpha)(-4)=9 \\
& \alpha^{2}=\frac{9}{4} \Rightarrow \alpha= \pm \frac{3}{2}
\end{aligned}
$$

If the sum of all the roots of the equation $e^{2 x}-11 e^{x}-45 e^{-x}+81 / 2=0$ is $\log _{e} p$, then $p$ is equal to 4.5

$$
\begin{aligned}
& e^{2 x}-11 e^{x}-\frac{45}{e^{x}}+\frac{81}{2}=0 \\
& \text { (JEE 2022) } \\
& e^{x}=t \\
& t_{1}=e^{x_{1}} \\
& x_{1}+x_{2}+x_{3}=\text { ? } \\
& 2 t\left(t^{2}-11 t-\frac{45}{t}+\frac{81}{2}=0\right) \\
& t_{2}=e^{x_{2}} \\
& t_{3}=e^{x_{3}}
\end{aligned}
$$

$$
\begin{aligned}
& e^{x_{1}} \cdot e^{x_{2}} \cdot e^{x_{3}}=45 \Rightarrow e^{x_{1}+x_{2}+x_{3}}=45 \Rightarrow x_{1}+x_{2}+x_{3}=\ln 45
\end{aligned}
$$ of $\mathrm{a}^{-2}+\mathrm{b}^{-2}+\mathrm{c}^{-2}$.



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