

# BOUNCE BACK 2.0



JEE MAINS & ADVANCED

ONE SHOT

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# TRIGONOMETRIC EQUATIONS

(8 Marks)

NISHANT VORA





# Nishant Vora

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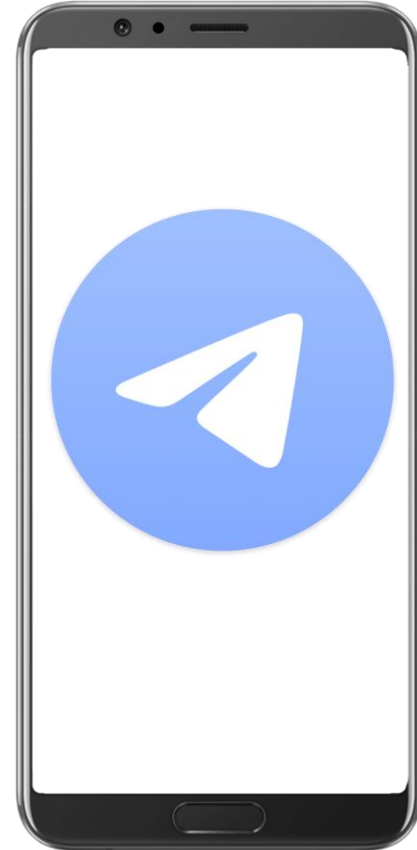
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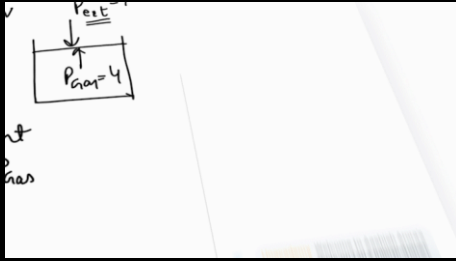
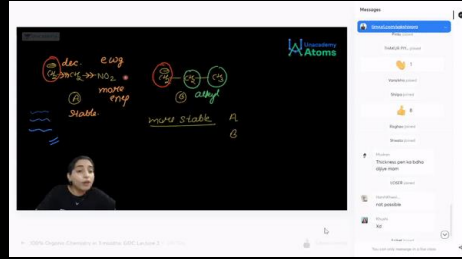
- [t.me/unacademyatoms](https://t.me/unacademyatoms)

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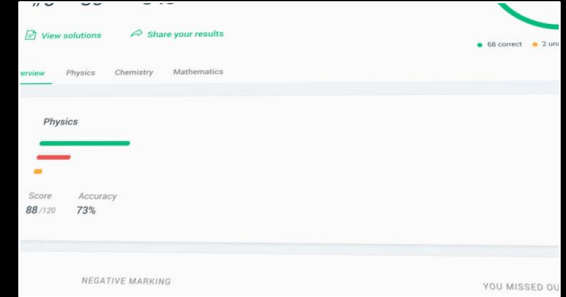


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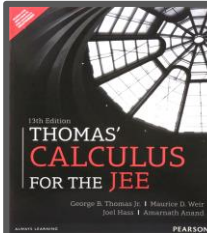
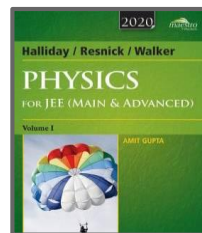
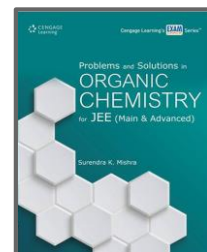
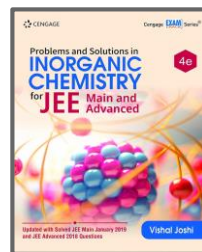
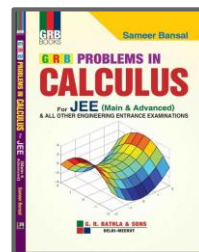
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Course of 12th syllabus Physics for JEE Aspirants 2022: Part - I

Lesson 1 • Apr 2, 2021 12:30 PM

D C Pandey

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# Trigonometric Equations



## Examples of Trigonometric Equations

1

$$\sin \theta = \frac{1}{2}$$

✓

$$\sin \theta = \frac{1}{2}$$

2

$$2 \sin x - \cos x = 3$$

# Solution of Trigonometric Equations

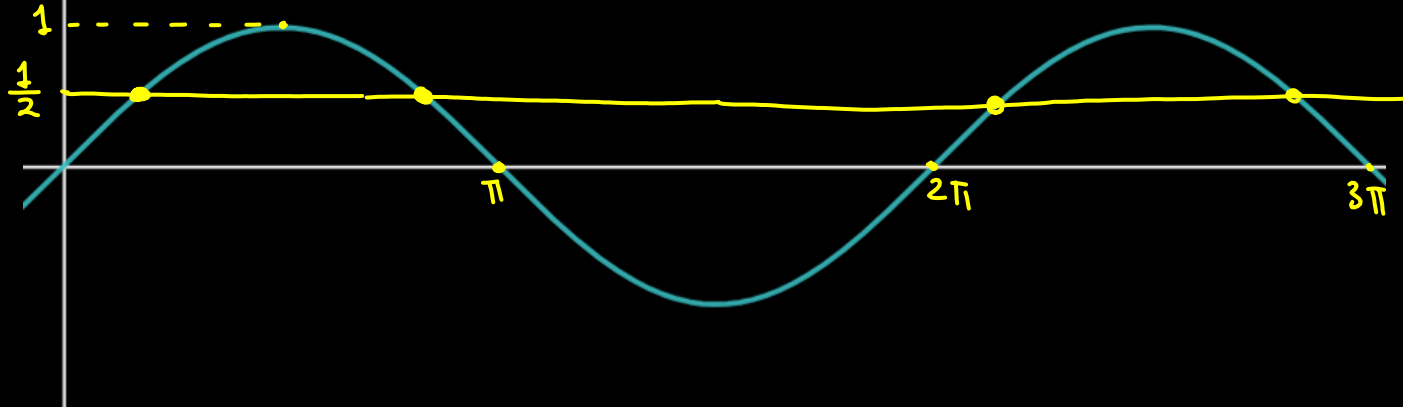
1

$$\sin \theta = \frac{1}{2}$$

$$\sin \theta = \frac{1}{2}$$

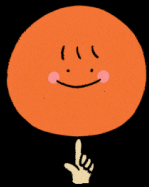
$$y = \sin \theta$$

$$y = \frac{1}{2}$$





# # Principal Solution



## Principal Solution

$$PS \in [0, 2\pi)$$

$$PS \in [0, 2\pi)$$



# Principal Solution - Shortcut Method

1

$$\sin \theta = \frac{1}{2}$$

#NVSTYLE

#NVStyle

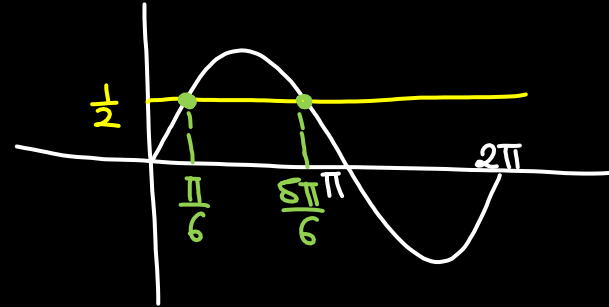
$$\underline{1-1} \quad \sin \alpha = \frac{1}{2} \Rightarrow \boxed{\alpha = \frac{\pi}{6}}$$

$$\underline{1-2} \quad \begin{array}{c|c} \textcircled{S} & \textcircled{A} \\ \hline T & C \end{array} \quad \underline{1^{\text{st}} \text{ and } 2^{\text{nd}}}$$

$$\underline{1-3} \quad \begin{array}{c|c} \pi - \alpha & \alpha \\ \hline \pi + \alpha & 2\pi - \alpha \end{array}$$

$$\frac{\pi}{6}, \pi - \frac{\pi}{6}$$

$$\Rightarrow \boxed{PS \Rightarrow \frac{\pi}{6}, \frac{5\pi}{6}}$$





# Principal Solution - Shortcut Method

#NVSTYLE

2

$$\sin x = -\frac{1}{2}$$

1-1  $\sin \alpha = \frac{1}{2}$   $\boxed{\alpha \Rightarrow \frac{\pi}{6}}$

1-2

S	A
(T)	(C)

 III and IV

1-3

$\pi - \alpha$	$\alpha$
$\pi + \alpha$	$2\pi - \alpha$

$$PS = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$$

$$\boxed{PS = \frac{7\pi}{6}, \frac{11\pi}{6}}$$

Ans Verify

$$\sin\left(\frac{7\pi}{6}\right)$$

$$= \sin\left(\pi + \frac{\pi}{6}\right)$$

$$= -\sin \frac{\pi}{6}$$

$$= -\frac{1}{2}$$

# Principal Solution - Shortcut Method

#NVSTYLE

3

$$\tan x = -\frac{1}{\sqrt{3}}$$

$$\underline{1-1} \quad \tan \alpha = \frac{1}{\sqrt{3}} \quad \boxed{\alpha = \frac{\pi}{6}}$$

$$\underline{1-2} \quad \begin{array}{c|c} \textcircled{S} & A \\ \hline T & \textcircled{C} \end{array} \quad \text{II and IV}$$

$$\underline{1-3} \quad \begin{array}{c|c} \pi - \alpha & \alpha \\ \hline \pi + \alpha & 2\pi - \alpha \end{array} \quad \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$$

$$\frac{5\pi}{6}, \frac{11\pi}{6}$$

# Principal Solution - Shortcut Method

#NVSTYLE

4

$$\operatorname{cosec} x = -2$$

$$\sin x = -\frac{1}{2}$$

$$\underline{\underline{1-1}} \quad \sin \alpha = \frac{1}{2} \quad \alpha = \frac{\pi}{6}$$

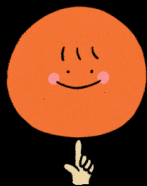
$$\underline{\underline{1-2}} \quad \begin{array}{c|c} S & A \\ \hline \textcircled{T} & \textcircled{C} \end{array}$$

$$\underline{\underline{1-3}} \quad PS \Rightarrow \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$$



# # General Solution





# General Solution

$$\sin \theta = \sin \alpha \quad \theta = n\pi + (-1)^n \alpha \quad n \in \mathbb{Z}$$

$$\cos \theta = \cos \alpha \quad \theta = 2n\pi \pm \alpha$$

$$\tan \theta = \tan \alpha \quad \theta = n\pi + \alpha$$

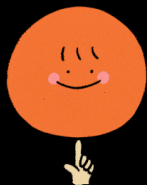


## General Solution

$$\left. \begin{aligned} \sin^2 \theta &= \sin^2 \alpha \\ \cos^2 \theta &= \cos^2 \alpha \\ \tan^2 \theta &= \tan^2 \alpha \end{aligned} \right\}$$

$$\boxed{\theta = n\pi \pm \alpha}$$

$$n \in \mathbb{Z}$$



## General solutions

$$\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha$$

$$\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha$$

$$\tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha$$

$$\sin^2 \theta = \sin^2 \alpha \Rightarrow \theta = n\pi \pm \alpha$$

$$\tan^2 \theta = \tan^2 \alpha \Rightarrow \theta = n\pi \pm \alpha$$

$$\cos^2 \theta = \cos^2 \alpha \Rightarrow \theta = n\pi \pm \alpha$$

$$\sin^2 \theta = \frac{1}{4}$$

$$\sin^2 \theta = \sin^2 \frac{\pi}{6}$$

$$\theta = n\pi \pm \frac{\pi}{6} \quad n \in \mathbb{Z}$$

# General Solutions - Examples

1

$$\sin x = -\frac{\sqrt{3}}{2}$$

Method (G.S)

1-1 P.S 1)

ii)

iii)

1-2 Use respective formula  
of G.S

S-1 1)  $\sin \alpha = \frac{\sqrt{3}}{2}$

$$\alpha = \frac{\pi}{3}$$

ii) 

S	A
T	C

PS  $\pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

PS  $\frac{4\pi}{3}, \frac{5\pi}{3}$

1-2  $\sin x = \sin \frac{4\pi}{3}$

$$x = n\pi + (-1)^n \frac{4\pi}{3} \quad n \in \mathbb{Z}$$



## General Solutions - Examples

2

$$\sin^2 x = \frac{1}{4}$$

$$\underline{\underline{S-1}} \quad \sin^2 x = \left(\frac{1}{2}\right)^2$$

$$\sin^2 x = \sin^2 \frac{\pi}{6}$$

$$\underline{\underline{S-2}} \quad \boxed{x = n\pi \pm \frac{\pi}{6}}$$

# General Solutions - Examples

3

$$\cot x = -1$$

$\cot x / \sec x / \operatorname{cosec} x$

$$\star \tan x = -1$$

Q-1 i)  $\tan \alpha = 1 \quad \alpha = \frac{\pi}{4}$

ii)  $\begin{array}{c|c} \text{S} & \text{A} \\ \hline \text{T} & \text{C} \end{array} \quad \text{II and IV}$

iii) PS  $\pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$   
 $\frac{3\pi}{4}, \frac{7\pi}{4}$

Q-2  $\tan x = \tan\left(\frac{3\pi}{4}\right)$

GS  $\boxed{x = n\pi + \frac{3\pi}{4}} \quad n \in \mathbb{Z}$





- ★ PSV
- ★ GSV
- ★ No of sol<sup>n</sup>

# Number of Solution

# Find Number of Solutions :

#NVSTYLE

1

$$\sin 4x = -\frac{1}{2} \text{ in } x \in [0, 2\pi]$$

$$\sin(\underline{4x}) = -\frac{1}{2}$$

$$x \in [0, 2\pi]$$

$$\boxed{\text{No of sol}^n = 8}$$

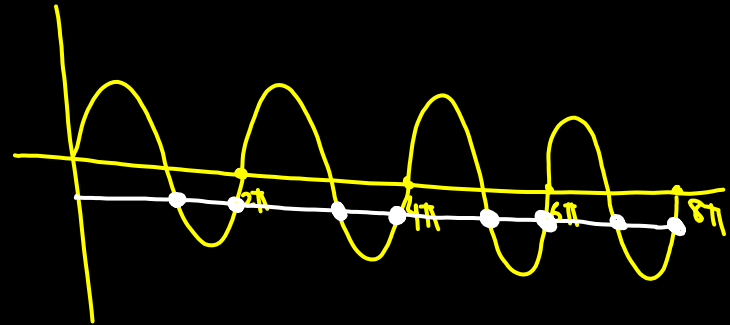
$$4x = \theta$$

$$\sin(4x) = -\frac{1}{2}$$

$$4x \in [0, 8\pi]$$

$$\sin(\theta) = -\frac{1}{2}$$

$$\theta \in [0, 8\pi]$$





## Find Number of Solutions :

2

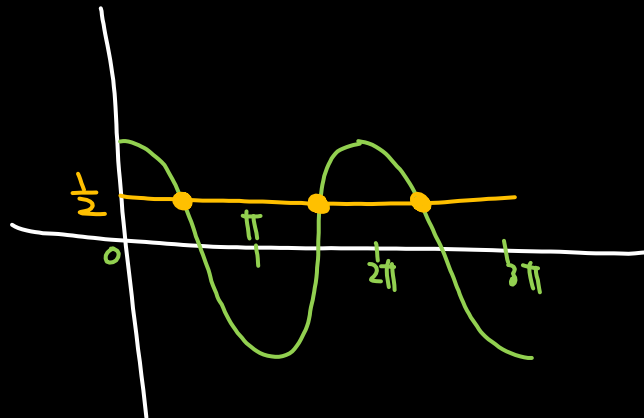
$$\sec 3x = 2 \text{ in } x \in [0, \pi]$$

$$\cos(3x) = \frac{1}{2} \quad x \in [0, \pi]$$

$$\cos(3x) = \frac{1}{2} \quad 3x \in [0, 3\pi]$$

$$\cos \theta = \frac{1}{2} \quad \theta \in [0, 3\pi]$$

No of sol<sup>n</sup> = 3



#NVSTYLE

Q

The number of distinct solutions of the equation

$$\frac{5}{4} \cos^2 2x + \boxed{\cos^4 x + \sin^4 x} + \cos^6 x + \sin^6 x = 2$$

in the interval  $[0, 2\pi]$  is

[JEE Adv. 2015]

$$\frac{5}{4} \cos^2 2x + \cancel{1} - 2 \sin^2 x \cos^2 x + \cancel{1} - 3 \sin^2 x \cos^2 x = \cancel{2}$$

$$\frac{5}{4} \cos^2 2x - \frac{5}{4} (4 \sin^2 x \cos^2 x) = 0$$

$$\frac{5}{4} (\cos^2 2x - \sin^2 2x) = 0$$

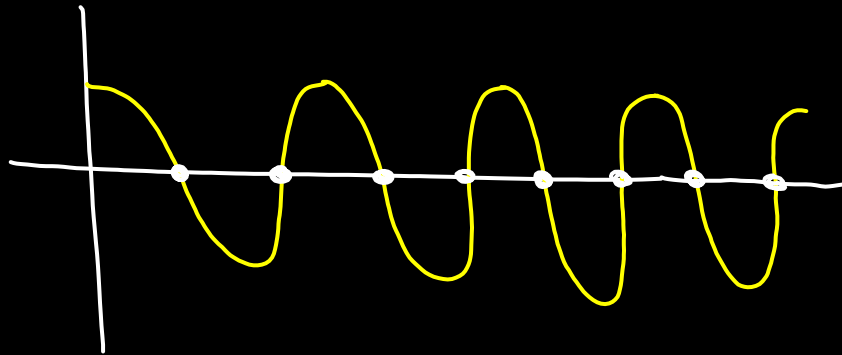
$$\sum_4 \cos 4x = 0$$

$$\Rightarrow \cos 4x = 0 \quad x \in [0, 2\pi]$$

$$\cos 4x = 0 \quad 4x \in [0, 8\pi]$$

$$\cos \theta = 0 \quad \theta \in [0, 8\pi]$$

No of Sol<sup>n</sup> = 8









PS ✓  
GS ✓  
No of Sol<sup>n</sup> ✓



# Type - 1

# Factorisation



## Type 1: Factorization/ Quadratic Form

$$\sin x / \cos x \rightarrow \boxed{Q.E}$$

↓  
factorize

↓  
Roots

— —

↓  
No. of sol<sup>n</sup> | Sum of sol<sup>n</sup>

Q

Solve  $(2 \sin x - \cos x)(1 + \cos x) = \sin^2 x$  in  $[0, 2\pi]$ .

$$(2 \sin x - \cos x)(1 + \cos x) = (1 - \cos x)(1 + \cos x)$$

$$2 \sin x - \cancel{\cos x} - 1 + \cancel{\cos x} = 0$$

$$1 + \cos x = 0$$

$$\cos x = -1$$

$$x = \pi$$

$$2 \sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$[0, 2\pi]$$

$$\frac{\pi}{6}, \pi - \frac{\pi}{6}$$

$$\rightarrow 3 \text{ sol}^n \cdot \left[ \pi, \frac{\pi}{6}, \frac{5\pi}{6} \right]$$

$$\text{Sum of sol}^n = 2\pi$$

★  
CUTE



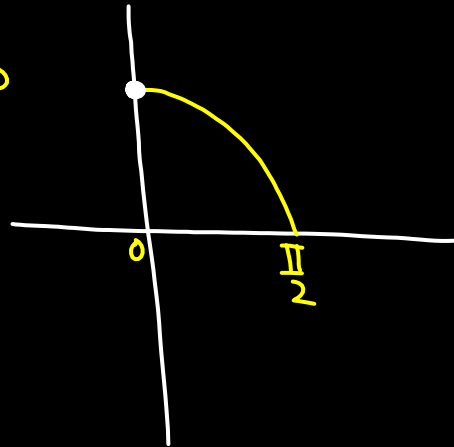
Q

If  $\sqrt{3} \cos^2 x = (\sqrt{3} - 1) \cos x + 1$ , the number of solutions of the given equation when  $x \in \left[0, \frac{\pi}{2}\right]$  is

$$\sqrt{3} \cos^2 x - \sqrt{3} \cos x + \cos x - 1 = 0$$

$$\sqrt{3} \cos x (\cos x - 1) + 1 (\cos x - 1) = 0$$

$$\begin{aligned} \cos x &= 1 \quad \text{OR} \quad \cos x = \frac{-1}{\sqrt{3}} \\ \boxed{1 + 0} &= \boxed{1} \end{aligned}$$



[JEE M 2021]



Q

Let  $4 + (-8) = -4$

$S = \{\theta \in [0, 2\pi] : 8^{2\sin^2 \theta} + 8^{2\cos^2 \theta} = 16\}$ . Then

[JEE M 2022]

$n(S)$  +  $\sum_{\theta \in S} \left( \sec\left(\frac{\pi}{4} + 2\theta\right) \operatorname{cosec}\left(\frac{\pi}{4} + 2\theta\right) \right)$  is

equal to :

$$8^{2\sin^2 \theta} + 8^{2-2\sin^2 \theta} = 16$$

$$8^{2\sin^2 \theta} = t$$

A. 0

B. -2

☒ C. -4

D. 12

$$\Rightarrow t + \frac{64}{t} = 16$$

$$\Rightarrow t^2 - 16t + 64 = 0$$

$$\Rightarrow (t-8)^2 = 0$$

$$8^{2\sin^2 \theta} = 8^1$$

$$\therefore \sin^2 \theta = \frac{1}{2}$$

$$\text{1st/2nd} \quad \sin \theta = \frac{+1}{\sqrt{2}}$$

$$\text{3rd/4th} \quad \sin \theta = \frac{-1}{\sqrt{2}}$$

$$\frac{2}{\cos \pi} + \frac{2}{\cos 3\pi} + \frac{2}{\cos 5\pi} + \frac{2}{\cos 7\pi}$$

$$S = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\} \quad \underline{n(S) = 4}$$

$$-2 \times 4 = \boxed{-8}$$

$$\sum_{\theta \in S} \frac{1 \times 2}{2 \cos\left(\frac{\pi}{4} + 2\theta\right) \sin\left(\frac{\pi}{4} + 2\theta\right)} \Rightarrow \sum_{\theta \in S} \frac{2}{\sin\left(\frac{\pi}{2} + 4\theta\right)}$$

$$\Rightarrow \sum_{\theta \in S} \left( \frac{2}{\cos 4\theta} \right)$$





If the sum of solutions of the system of equations  $2\sin^2\theta - \cos 2\theta = 0$  and  $2\cos^2\theta + 3\sin\theta = 0$  in the interval  $[0, 2\pi]$ , then  $k$  is equal to\_\_.

[JEE M 2022]

$$2\sin^2\theta = \cos 2\theta$$

$$2\sin^2\theta = 1 - 2\sin^2\theta$$

$$\sin^2\theta = \frac{1}{4}$$

$$\sin\theta = \frac{1}{2} \text{ or } -\frac{1}{2}$$

$$2\cos^2\theta + 3\sin\theta = 0$$

$$2(1 - \sin^2\theta) + 3\sin\theta = 0$$

$$2\sin^2\theta - 3\sin\theta - 2 = 0$$

$$2\sin^2\theta - 4\sin\theta + \sin\theta - 2 = 0$$

$$(2\sin\theta + 1)(\sin\theta - 2) = 0$$

$$\sin\theta = -\frac{1}{2} \text{ or } 2$$

$$\sin \theta = -\frac{1}{2} \quad [0, 2\pi]$$

$$\underline{1-1} \quad \sin \alpha = \frac{1}{2} \quad \alpha = \frac{\pi}{6}$$

$$\underline{1-2} \quad \begin{array}{c|c} S & A \\ \hline T & C \end{array} \quad 3^{\text{rd}} \text{ and } 4^{\text{th}}$$

$$\underline{1-3} \quad \text{PS: } \pi + \frac{\pi}{6} \text{ and } 2\pi - \frac{\pi}{6}$$

$$\boxed{\frac{7\pi}{6}, \frac{11\pi}{6}}$$

$$\text{sum} = 3\pi$$

Q

If  $S = \{\theta \in (0, 2\pi) : 7 \cos^2\theta - 3 \sin^2\theta - 2 \cos^2 2\theta = 2\}$ . Then, the sum of roots of all the equations  $x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6 \sin^2\theta = 0$   $\theta \in S$ , is \_

[JEE M 2022]

$$\Rightarrow 7\left(\frac{1 + \cos 2\theta}{2}\right) - 3\left(\frac{1 - \cos 2\theta}{2}\right) - 2 \cos^2 2\theta = 2$$

$$\Rightarrow 7 + 5 \cos 2\theta - 2 \cos^2 2\theta = 7$$

$$\Rightarrow \cos 2\theta (5 - 2 \cos 2\theta) = 0$$

$$\Rightarrow \boxed{\cos 2\theta = 0} \text{ OR } \cancel{\cos 2\theta = \frac{5}{2}}$$

$$\cos 2\theta = 0 \quad \theta \in (0, 2\pi)$$

$$\cos(2\theta) = 0 \quad 2\theta \in (0, 4\pi)$$

$$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$S = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$

$$\text{Sum of Root} = \boxed{2(\tan^2 \theta + \cot^2 \theta)}$$

$$\Rightarrow 4 + 4 + 4 + 4$$

$$\Rightarrow 16$$

Q

Let

$$S = \left\{ \theta \in [-\pi, \pi] - \left\{ \pm \frac{\pi}{2} \right\} : \sin \theta \tan \theta + \tan \theta = \sin 2\theta \right\}.$$

If  $T = \sum_{\theta \in S} \cos 2\theta$ , then  $\underline{T} + \underline{n(S)}$  is equal

[JEE M 2022]

A.  $7 + \sqrt{3}$

✓ B. 9

C.  $8 + \sqrt{3}$

D. 10

$$\cancel{\tan \theta} (\sin \theta + 1) = \frac{2 \cancel{\tan \theta}}{1 + \cancel{\tan^2 \theta}}$$

$$\tan \theta = 0$$

$$\sin \theta + 1 = 2 \cos^2 \theta$$

$$\sin \theta + 1 = 2(1 - \sin^2 \theta)$$

$$2 \sin^2 \theta + \sin \theta - 1 = 0$$

$$2 \sin^2 \theta + 2 \sin \theta - \sin \theta - 1 = 0$$

$$(2 \sin \theta - 1)(\sin \theta + 1) = 0$$

$$\underline{[-\pi, \pi] - \left\{ \pm \frac{\pi}{2} \right\}}$$

$$\tan \theta = 0$$

$$\underline{-\pi, 0, \pi}$$

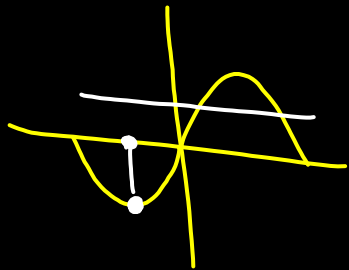
$$\sin \theta = \frac{1}{2}$$

$$\underline{\underline{\frac{\pi}{6}, \frac{5\pi}{6}}}$$

$$\sin \theta = -1$$

No sol<sup>n</sup>

$$S = \left\{ -\pi, 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6} \right\}$$



$$T = \sum_{\theta \in S} \cos 2\theta$$

$$= \cos(-2\pi) + \cos 0 + \cos 2\pi + \cos \frac{\pi}{3} + \cos \frac{5\pi}{3}$$

$$= 1 + 1 + 1 + \frac{1}{2} + \frac{1}{2}$$

$$= 4$$



The number of elements in the set  $S = \{\theta \in [-4\pi, 4\pi] : \underline{3 \cos^2 2\theta + 6 \cos 2\theta - 10 \cos^2 \theta + 5 = 0}\}$  is 32.

[JEE M 2022]

$$3 \cos^2 2\theta + 6 \cos 2\theta - 5(2 \cos^2 \theta) + 5 = 0$$

$$3 \cos^2 2\theta + \underline{6 \cos 2\theta} - 5(1 + \cos 2\theta) + \cancel{5} = 0$$

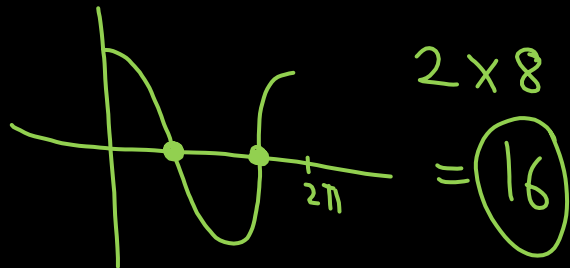
$$3 \cos^2 2\theta + \cos 2\theta = 0$$

$$\cos 2\theta (3 \cos 2\theta + 1) = 0$$

$$\cos 2\theta = 0 \quad \cos 2\theta = -\frac{1}{3}$$

$$\cos 2\theta = 0 \quad \theta \in [-4\pi, 4\pi]$$

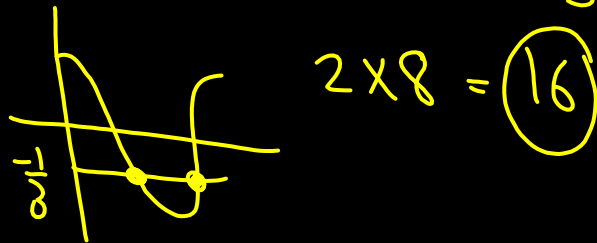
$$\cos x = 0 \quad x \in [-8\pi, 8\pi]$$



$$\cos 2\theta = \frac{1}{3} \quad \theta \in [-4\pi, 4\pi]$$

$$\cos 2\theta = -\frac{1}{3} \quad 2\theta \in [-8\pi, 8\pi]$$

$$\cos x = -\frac{1}{3} \quad x \in [-8\pi, 8\pi]$$







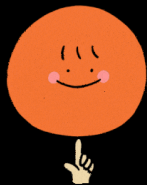
$$a \sin x + b \cos x = c$$

??

# Type - 2



$$a \sin x + b \cos x = c$$



## Type 2

$$a \sin x + b \cos x = c$$

Method

$\Rightarrow$  divide by  $\sqrt{a^2 + b^2}$  both sides.

Q

Find general solution of  $\sin x + \cos x = \sqrt{2}$ .

$$\left\{ \frac{1}{\sqrt{2}} \right\} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{\sqrt{2}}{\sqrt{2}}$$

$$\sin \frac{\pi}{4} \sin x + \cos \frac{\pi}{4} \cos x = 1$$

$$\# \cos\left(x - \frac{\pi}{4}\right) = \cos 0$$

$$x - \frac{\pi}{4} = 2h\pi \pm 0$$

$$\boxed{x = 2h\pi + \frac{\pi}{4}} \quad n \in \mathbb{Z}$$

$$\sqrt{(1)^2 + (1)^2} = \sqrt{2}$$

$$\underline{2h\pi \pm \alpha}$$

$$\underline{n\pi + (-1)^n \alpha}$$

Q

Find general solution of  $\sqrt{3} \cos x + \sin x = 2$ .

$$\boxed{\frac{\sqrt{3}}{2}} \cos x + \frac{1}{2} \sin x = \frac{2}{2}$$

$$\sqrt{(\sqrt{3})^2 + 1^2} = 2$$

$$\cos \frac{\pi}{6} \cos x + \sin \frac{\pi}{6} \sin x = 1$$

$$\cos \left( x - \frac{\pi}{6} \right) = \cos 0$$

$$x - \frac{\pi}{6} = 2n\pi$$

$$\boxed{x = 2n\pi + \frac{\pi}{6}}$$

Q

Find the general solutions of equation  $\sin x + \cos x = 3/2$

$$\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{3}{2\sqrt{2}}$$

$$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \frac{3}{2\sqrt{2}}$$

No sol<sup>n</sup>

$$\cos\left(x - \frac{\pi}{4}\right) = 1.5m$$

$$\frac{\sqrt{9}}{\sqrt{8}} = \sqrt{\frac{9}{8}}$$

$$1.5m$$

Q

The number of integral values of 'k' for which the equation

$3 \sin x + 4 \cos x = k + 1$  has a solution,  $k \in R$  is

$$[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$$

$$[-5, 5] \quad k+1$$

$$-5 \leq k+1 \leq 5$$

$$-6 \leq k \leq 4$$

$$-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4$$

11 Values

[JEE M 2021]

Concept - "sol" Exist Karla hai"

$$\sin x = 4$$

$[-1, 1]$

No soln

$$\sin x = \frac{1}{2}$$

$[-1, 1]$

$\frac{\sqrt{3}}{2}$

$\frac{1}{3}$

$\frac{2}{3}$

$$\sin x = k$$
$$-1 \leq k \leq 1$$



Consider the following lists:

### List-I

- (I)  $\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$
- ~~(II)~~  $\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3} \tan 3x = 1\right\}$
- (III)  $\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2 \cos(2x) = \sqrt{3}\right\}$
- (IV)  $\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$

### List-II

- (P) has two elements
- (Q) has three elements
- (R) has four elements
- (S) has five elements
- (T) has six elements

$$\begin{aligned} \tan\left(-\frac{5\pi}{6}\right) &= -\tan\frac{5\pi}{6} \\ &= -\tan\left(\pi - \frac{\pi}{6}\right) \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$

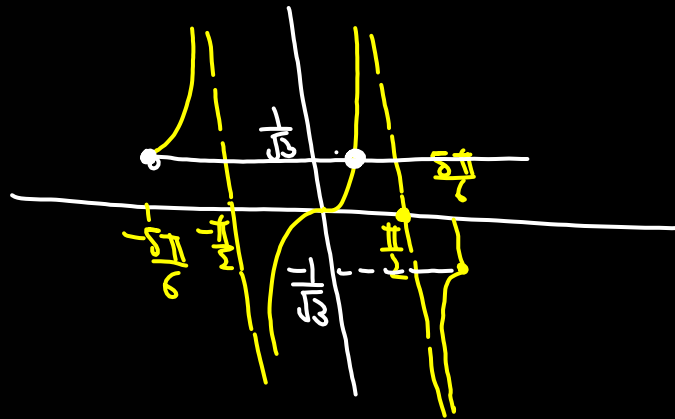
[JEE Adv. 2022]

$$\tan 3x = \frac{1}{\sqrt{3}} \quad x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right]$$

$$\tan(\theta) = \frac{1}{\sqrt{3}} \quad \theta \in \left[-\frac{5\pi}{6}, \frac{5\pi}{6}\right]$$

The correct option is:

- ~~(A)~~ (I)  $\rightarrow$  (P); (II)  $\rightarrow$  (S); (III)  $\rightarrow$  (P); (IV)  $\rightarrow$  (S)
- (B) (I)  $\rightarrow$  (P); (II)  $\rightarrow$  (P); (III)  $\rightarrow$  (T); (IV)  $\rightarrow$  (R)
- (C) (I)  $\rightarrow$  (Q); (II)  $\rightarrow$  (P); (III)  $\rightarrow$  (T); (IV)  $\rightarrow$  (S)
- ~~(D)~~ (I)  $\rightarrow$  (Q); (II)  $\rightarrow$  (S); (III)  $\rightarrow$  (P); (IV)  $\rightarrow$  (R)



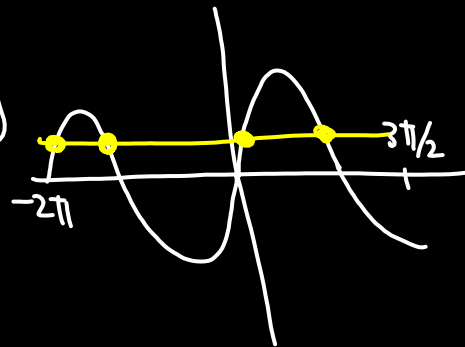


$$-\frac{7\pi}{4} - \frac{\pi}{4}, \frac{7\pi}{4} - \frac{\pi}{4}$$

$$\frac{1}{\sqrt{2}} \sin x - \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}} \quad x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right]$$

$$\sin\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \quad \left(x - \frac{\pi}{4}\right) \in \left[-2\pi, \frac{3\pi}{2}\right]$$

$$\sin(\theta) = \frac{1}{\sqrt{2}} \quad \theta \in \left[-2\pi, \frac{3\pi}{2}\right]$$







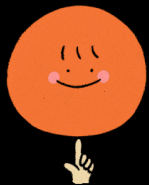


① Q.E - factorise

②  $a \sin x + b \cos x = c$

# # Type - 3

(Convert Sum to Product)



## Type 3

Convert Sum to Product

Q

If  $0 \leq x < \frac{\pi}{2}$ , then the number of values of  $x$  for which

$\sin x - \sin 2x + \sin 3x = 0$  is  $x \in \left[0, \frac{\pi}{2}\right)$

- A. 3  
B. 1  
C. 4  
✓ D. 2

$$\sin x + \sin 3x - \sin 2x = 0$$

$$2 \sin 2x \cos x - \sin 2x = 0$$

$$\sin 2x (2 \cos x - 1) = 0$$

$$\sin 2x = 0 \text{ OR } \cos x = \frac{1}{2}$$

[JEE M 2019]

$$\sin 2x = 0 \quad x \in \left[0, \frac{\pi}{2}\right)$$

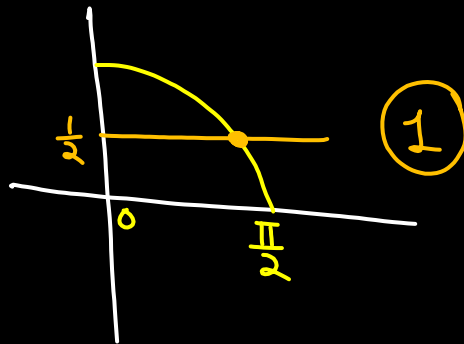
$$\sin 2x = 0 \quad 2x \in [0, \pi)$$

$$\sin \theta = 0 \quad \theta \in [0, \pi)$$

$$\boxed{\theta = 0}$$

$$1 + 1 = \boxed{2}$$

$$\cos x = \frac{1}{2} \quad x \in \left[0, \frac{\pi}{2}\right)$$



Q

If the sum of values of  $x$  in  $[0, 2\pi]$ , for which  $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$ , is equal to

- A.  $8\pi$   
 B.  $11\pi$   
 C.  $12\pi$   
 ✓ D.  $9\pi$

$$\underline{2 \sin\left(\frac{5x}{2}\right) \cos\left(\frac{3x}{2}\right)} + \underline{2 \sin\left(\frac{5x}{2}\right) \cos\left(\frac{x}{2}\right)} = 0$$

D

$$\frac{C-D}{2} = \frac{x-4x}{2}$$

$$= \left(\frac{-3x}{2}\right)$$

$$2 \sin\left(\frac{5x}{2}\right) \left\{ \cos \frac{3x}{2} + \cos \frac{x}{2} \right\} = 0$$

$$\underset{\checkmark}{2 \sin\left(\frac{5x}{2}\right)} \left\{ \underset{\checkmark}{2 \cos x} \underset{\checkmark}{\cos \frac{x}{2}} \right\} = 0$$

[JEE M 2021]



$$x \in [0, 2\pi]$$

$$\sin \frac{5x}{2} = 0$$

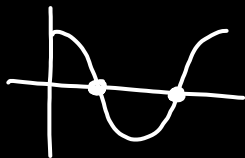
$$\sin\left(\frac{5x}{2}\right) = 0 \quad \frac{5x}{2} \in [0, 5\pi]$$

$$\frac{5x}{2} = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$$

$$x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi$$

$$x \in (0, 2\pi]$$

$$\cos x = 0$$



$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\text{Sum of Sol}^n \Rightarrow 9\pi$$

$$\cos \frac{x}{2} = 0 \quad x \in [0, 2\pi]$$

$$\cos\left(\frac{x}{2}\right) = 0 \quad \frac{x}{2} \in [0, \pi]$$

$$\frac{x}{2} = \frac{\pi}{2}$$

$$x = \pi$$



Q

The positive integer value of  $n > 3$  satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)} \text{ is}$$

Sum  $\Rightarrow$  Product

[JEE Adv. 2011]

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} - \frac{1}{\sin\left(\frac{3\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)}$$

$$\frac{\sin\left(\frac{3\pi}{n}\right) - \sin\left(\frac{\pi}{n}\right)}{\sin\left(\frac{\pi}{n}\right) \sin\left(\frac{3\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)}$$

$$\frac{2 \cos\left(\frac{2\pi}{n}\right) \cancel{\sin\left(\frac{\pi}{n}\right)}}{\cancel{\sin\left(\frac{\pi}{n}\right)} \sin\left(\frac{3\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)}$$

$$2 \cos\left(\frac{2\pi}{n}\right) \sin\left(\frac{2\pi}{n}\right) = \sin\left(\frac{3\pi}{n}\right)$$

$$\sin\left(\frac{4\pi}{n}\right) = \sin\left(\frac{3\pi}{n}\right)$$

ans.:-

$$\boxed{n = 7}$$

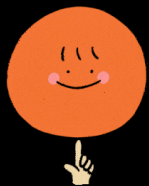
$$\sin\left(\frac{4\pi}{7}\right) = \sin\left(\frac{3\pi}{7}\right)$$

$$\star \star \boxed{\sin \theta = \sin(\pi - \theta)}$$



# Type - 4

## (Convert Product to Sum)



## Type 4

Convert Product into Sum

Q

Number of solutions of the trigonometric equation in  $[0, \pi]$ ,

$$\sin 3\theta = 4 \sin \theta \cdot \sin 2\theta \cdot \sin 4\theta$$

$$\sin(A-B) \cdot \sin(A+B) = \sin^2 A - \sin^2 B$$

A. 4

B. 6

✓ C. 8

D. 10

$$\Rightarrow \sin 3\theta = 4 \sin \theta \sin(3\theta - \theta) \sin(3\theta + \theta)$$

$$\Rightarrow \sin 3\theta = 4 \sin \theta (\sin^2 3\theta - \sin^2 \theta)$$

$$\Rightarrow 3 \sin \theta - 4 \sin^3 \theta = 4 \sin \theta \sin^2 3\theta - 4 \sin^3 \theta$$

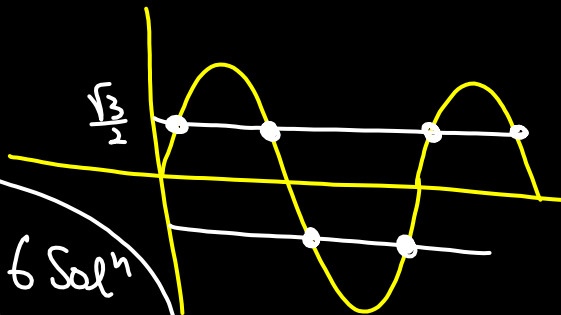
$$\Rightarrow 3 \sin \theta = 4 \sin \theta \sin^2 3\theta$$

$$\sin \theta = 0 \quad 3 = 4 \sin^2 3\theta \quad \theta \in [0, \pi]$$

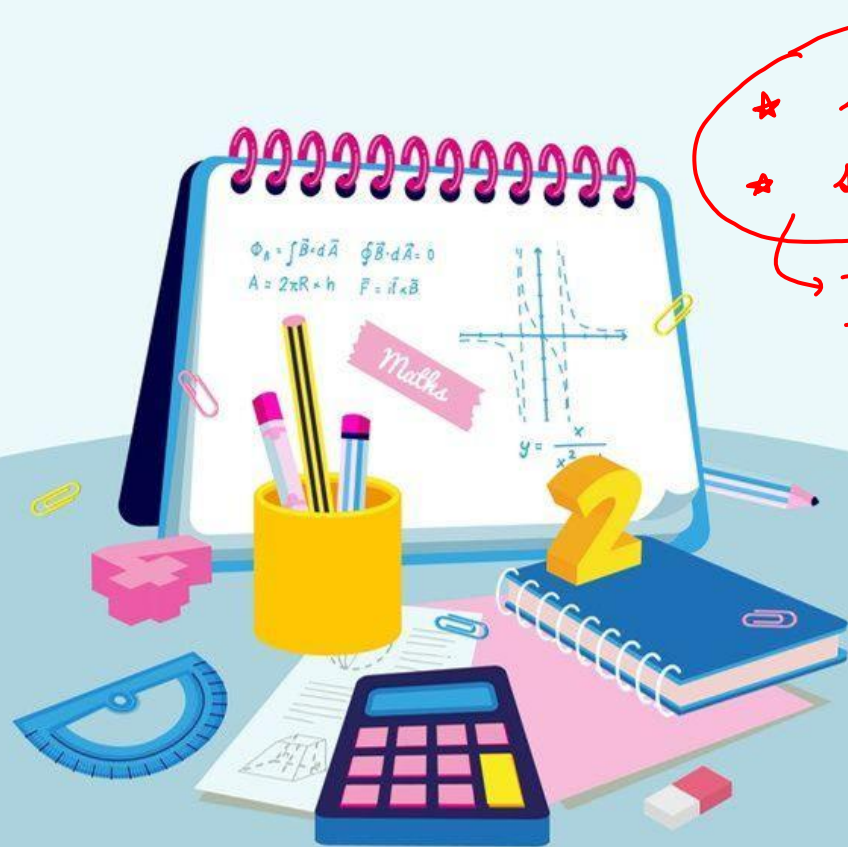
$$\left. \begin{array}{l} \sin \theta = 0 \\ \theta \in [0, \pi] \end{array} \right\} \begin{array}{l} \sin 3\theta = \pm \frac{\sqrt{3}}{2} \quad 3\theta \in [0, 3\pi] \\ \sin \pi = \pm \frac{\sqrt{3}}{2} \quad \pi \in [0, 3\pi] \end{array}$$

$$\theta = 0, \pi$$

$$2 \text{ Sol}^n + 6 \text{ Sol}^n$$







$$\star \sin x \pm \cos x \quad \checkmark$$
$$\star \sin x \cos x \quad \checkmark$$

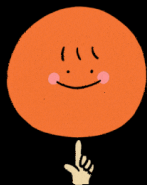
→ Type - 5

$$\sin x + \cos x = \sin x \cos x + 2$$

→ Method

# Type - 5

$$f(\sin x \pm \cos x, \sin x \cos x)$$



## Type 5

(i) Equations of the form  $P(\sin x \pm \cos x, \sin x \cdot \cos x) = 0$ , can be solved by the substituting  $\cos x \pm \sin x = t$

(ii) Many equations can be solved by introducing a new variable  
e.g. consider the equation  $\sin^4 2x + \cos^4 2x = \sin 2x \cdot \cos 2x$

$$\underline{\sin x + \cos x} = \underline{\sin x \cos x} + 2$$

Method  $\sin x + \cos x = t$

$$\Rightarrow (\sin x + \cos x)^2 = t^2$$

$$\Rightarrow 1 + 2 \boxed{\sin x \cos x} = t^2$$

$$\Rightarrow \boxed{\sin x \cos x = \frac{t^2 - 1}{2}}$$

Q.E

$$t = \frac{t^2 - 1}{2} + 2$$

Q

Find general value of  $x$  satisfying the equation

$$\sin^4 2x + \cos^4 2x = \sin 2x \cos 2x.$$

$$2(1 - 2\sin^2 2x \cos^2 2x) = \sin 2x \cos 2x$$

$$\Rightarrow 2 - \sin^2 4x = \sin 4x$$

$$\Rightarrow \sin^2 4x + \sin 4x - 2 = 0$$

$$\Rightarrow (\sin 4x + 2)(\sin 4x - 1) = 0$$

$$\cancel{\sin 4x = -2} \quad \sin 4x = 1$$

$$\sin 4x = \sin \frac{\pi}{2}$$

$$x = \frac{n\pi}{4} + (-1)^n \frac{\pi}{8}$$





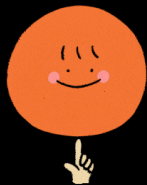
$$\underline{LHS} = \underline{RHS}$$
$$[-2, 2] \quad [2, 8]$$

$$\boxed{LHS=2 \quad RHS=2}$$



# Type – 6

## Using Range of Functions



## Type 6

Solving equations with the use of boundedness of the function.

Remember :-

$$\underline{-1 \leq \sin x \leq 1}, \underline{-1 \leq \cos x \leq 1}, \underline{\tan x \in \mathbb{R}, \cot x \in \mathbb{R}}.$$

$$\underline{|\operatorname{cosec} x| \geq 1}, \underline{|\sec x| \geq 1}.$$

$$\left. \begin{array}{l} \sec x \\ \operatorname{cosec} x \end{array} \right\} (-\infty, -1] \cup [1, \infty)$$

Q

Most GSSolve for x:  $\cos x + \cos 2x + \cos 3x = 3$ 

[-1, 1]

[-1, 1]

[-1, 1]

 $n \in \mathbb{Z}$ 

$$\cos x = 1$$

$$\cos 2x = 1$$

$$\cos 3x = 1$$

~~$$x = 2n\pi$$~~

$$2x = 2n\pi$$

$$3x = 2n\pi$$

~~$$\{0, 2\pi, 4\pi, 6\pi, \dots\}$$~~

$$\therefore x = n\pi$$

$$x = \frac{2n\pi}{3}$$

$$\{0, \pi, 2\pi, 3\pi, 4\pi, \dots\}$$

$$\{0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi, \frac{8\pi}{3}, \dots\}$$

$$GS \in \{n\pi\} \cup \{\frac{2n\pi}{3}\}$$



Q

All the pairs  $(x, y)$  that satisfy the inequality

$$2\sqrt{\sin^2 x - 2\sin x + 5} \leq \frac{1}{\sin^2 y} \leq 1$$

(1)  $2|\sin x| = 3\sin y$

(2)  $2\sin x = \sin y$

(3)  $\sin x = 2\sin y$

(4)  $\sin x = |\sin y|$

(D')

$\sin x = -1$

$$\sqrt{\sin^2 x - 2\sin x + 5} \leq 2\sin^2 y$$

$$\sqrt{(\cancel{\sin x} - 1)^2 + 4} \leq \sqrt{2\sin^2 y}$$

$$[2, 2\sqrt{2}] \quad [0, 2]$$

$\sin x = 1$

$\sin^2 y = 1$

$\sin y = \pm 1$

$\sin x = |\sin y|$

[JEE M 2019]



Q

The number of solutions of  $\sin^7 x + \cos^7 x = 1$ ,  $x \in [0, 4\pi]$  is equal to

- A. 11  
B. 7  
✓ C. 5  
D. 9

5 sol<sup>n</sup>

$$\sin^7 x + \cos^7 x = 1$$

$\sin x$	$\cos x$
1	0
0	1

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$x = 0, 2\pi, 4\pi$$

[JEE M 2021]



Q

For  $x \in (0, \pi)$ , the equation  $\sin x + 2\sin 2x - \sin 3x = 3$  has

- (a) infinitely many solutions
  - (b) three solutions
  - (c) one solution
  - ✓ (d) no solution
- $-2 \cos 2x \sin x + 2 \sin 2x = 3$

[JEE Adv. 2014]

D

$$-2 \cos 2x \sin x + 4 \sin x \cos x = 3$$

$$\sin x (-2 \cos 2x + 4 \cos x) = 3$$

$$-2(2 \cos^2 x - 1) + 4 \cos x = \frac{3}{\sin x}$$

$$\underbrace{-1 - 4\cos^2 x + 4\cos x + 3}_{[-6, 3]} = 3 \operatorname{cosec} x$$

$$\boxed{3 - (2\cos x - 1)^2} = \boxed{3 \operatorname{cosec} x}$$

$$[-6, 3]$$

$$[3, \infty)$$

$$\cos x = -1$$

$$\cos x = \frac{1}{2}$$

No Sol<sup>n</sup>

$$\boxed{\cos x = \frac{1}{2}}$$

$$\operatorname{cosec} x = 1$$

$$\boxed{\sin x = 1}$$

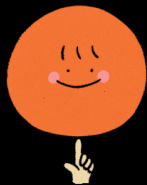


(Root wale sawal)



# Type - 7

$$\underline{f(x) = \sqrt{\varphi(x)}}$$



## Type 7

Solution of trigonometric equation of the form  $f(x) = \sqrt{\varphi(x)}$

(i)  $f(x) \geq 0$ ,  $\varphi(x) \geq 0$

(ii)  $f^2(x) = \varphi(x)$

★ Jab bhi sq both sides

⇓

fake sol<sup>n</sup>

⇓

Reject





Solve for  $x$ ,  $\sin x = \sqrt{1 - \cos x}$  in  $x \in [0, 2\pi]$

$$\sin^2 x = 1 - \cos x$$

$$(1 - \cancel{\cos x})(1 + \cos x) = (1 - \cancel{\cos x})$$

$$\cos x = 1$$

$$x = 0, 2\pi$$

$$1 + \cos x = 1$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sin x = \sqrt{1 - \cos x}$$

$x = 0$	$0 = \sqrt{1-1}$ ✓
$x = 2\pi$	$0 = \sqrt{1-1}$ ✓
$x = \frac{\pi}{2}$	$1 = \sqrt{1-0}$ ✓
<del><math>x = \frac{3\pi}{2}</math></del>	<del><math>-1 = \sqrt{1-0}</math></del> ✗





# ★ Type – 8

## Log wale Questions



The number of distinct solutions of the equation,

$$\log_{\frac{1}{2}} |\sin x| = 2 - \log_{\frac{1}{2}} |\cos x| \text{ in the interval } [0, 2\pi], \text{ is } \underline{\hspace{2cm}}$$

$$\log_{\frac{1}{2}} |\sin x| + \log_{\frac{1}{2}} |\cos x| = 2$$

$$\log_{\left(\frac{1}{2}\right)} |\sin x| |\cos x| = 2$$

$$\underbrace{|a| |b| = |ab|}$$

$$|\sin x \cos x| = \frac{1}{4} \times 2$$

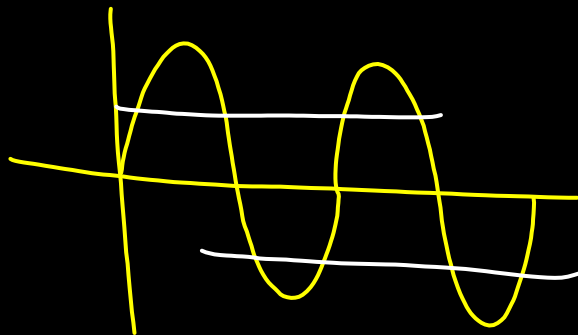
$$|\sin 2x| = \frac{1}{2}$$

[JEE M 2020]

$$\sin 2x = \pm \frac{1}{2} \quad x \in [0, 2\pi]$$

$$\sin(2x) = \pm \frac{1}{2} \quad 2x \in [0, 4\pi]$$

$$\star \sin(\theta) = \pm \frac{1}{2} \quad \theta \in [0, 4\pi]$$



(8)

Q

If for  $x \in (0, \frac{\pi}{2})$ ,  $\log_{10} \sin x + \log_{10} \cos x = -1$  and

$\log_{10}(\sin x + \cos x) = \left(\frac{1}{2}\right)(\log_{10} n - 1)$   $n > 0$  then the value of  $n$  is

- A. 20
- ☒ B. 12
- C. 9
- D. 16

$$\log_{10} (\sin x \cos x) = -1$$

$$\sin x \cos x = 10^{-1} = \frac{1}{10} \quad \text{--- (1)}$$

$$\log_{10} (\sin x + \cos x)^2 = \log_{10} n - \log_{10} 10$$

$$\log_{10} (1 + 2 \sin x \cos x) = \log_{10} \left( \frac{n}{10} \right)$$

[JEE M 2021]

$$1 + 2 \times \frac{1}{10} = \frac{n}{10}$$

$$\cdot \boxed{n = 12}$$





Trig + algebra

$$\tan x = x - 10$$

$$y = \tan x$$

$$y = x - 10$$

P.O.I

#

no. of  
sol<sup>n</sup>

# Type - 9

## Graph wale Questions

Q

The number of solutions of equation  $x + 2 \tan x = \frac{\pi}{2}$  in the interval  $[0, 2\pi]$  is :

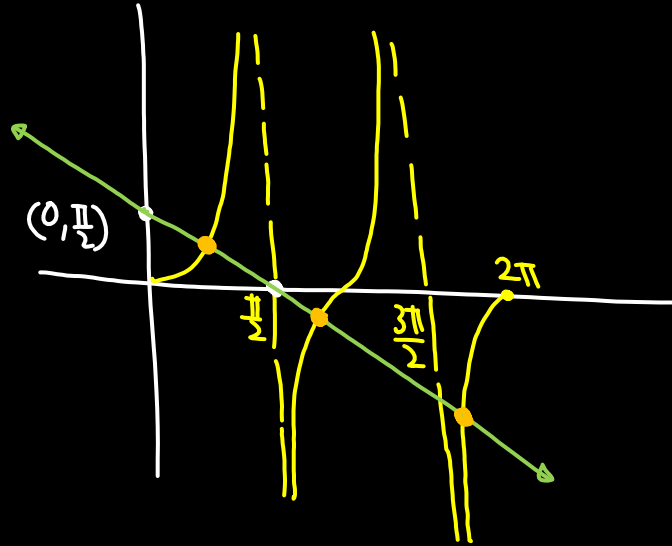
- A. 3  
B. 4  
C. 2  
D. 5

$$y = 2 \tan x$$

$$y = \frac{\pi}{2} - x$$

$x$	$y$
0	$\pi/2$
$\pi/2$	0

$$2 \tan x = \frac{\pi}{2} - x$$



[JEE M 2021]



Q

The number of solutions of  $|\cos x| = \sin x$ , such that  $-4\pi \leq x \leq 4\pi$  is

A. 4

B. 6

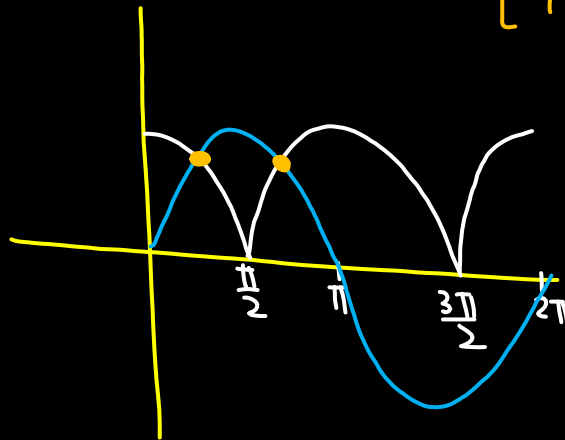
☒ C. 8

D. 12

 $[0, 2\pi] \rightarrow 2 \text{ sol}^n$  [JEE M 2022] $[-4\pi, 4\pi] \rightarrow 2 \times 4$  $= \textcircled{8} \text{ sol}^n$ 

$$y = |\cos x|$$

$$y = \sin x$$



Q

The number of solutions of the equation  $2\theta - \cos^2\theta + \sqrt{2} = 0$  in  $\mathbb{R}$  is equal to 1.

[JEE M 2022]

$$y = 2\theta + \sqrt{2}$$

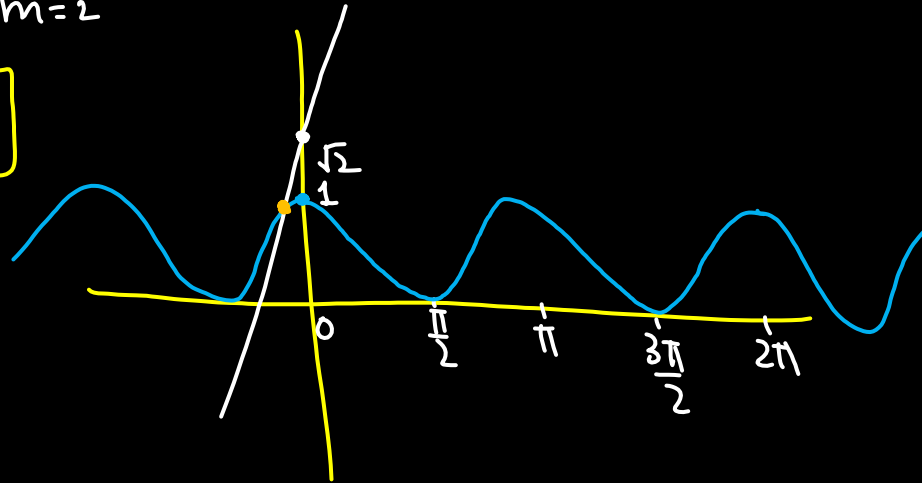
$$(0, \sqrt{2}) \quad y = 2x + \sqrt{2} \quad m=2$$

$$2\theta + \sqrt{2} = \cos^2\theta$$

$$\theta \in (-\infty, \infty)$$

$$y = \cos^2\theta$$

$\theta$	$y$
0	1
$\frac{\pi}{2}$	0
$\pi$	1
$\frac{3\pi}{2}$	0
$2\pi$	1



Q

The number of solutions of the equation  $\sin x = \cos^2 x$  in the interval  $(0, 10)$  is \_.

$$\sqrt{5} \approx 2.2$$

$$\sqrt{3} \approx 1.7$$

[JEE M 2022]

M-1

$$\sin x = \cos^2 x$$

④

$$\sin x = 1 - \sin^2 x$$

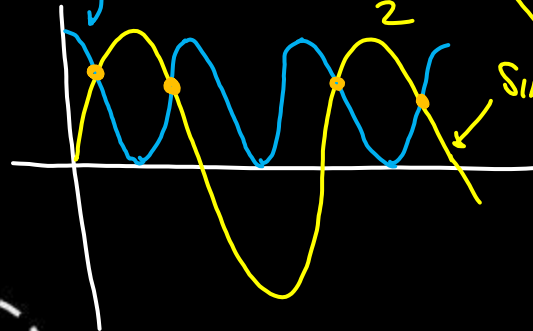
$$\sin^2 x + \sin x - 1 = 0$$

$\cos^2 x$

$$\sin x = \frac{-1 \pm \sqrt{5}}{2}$$

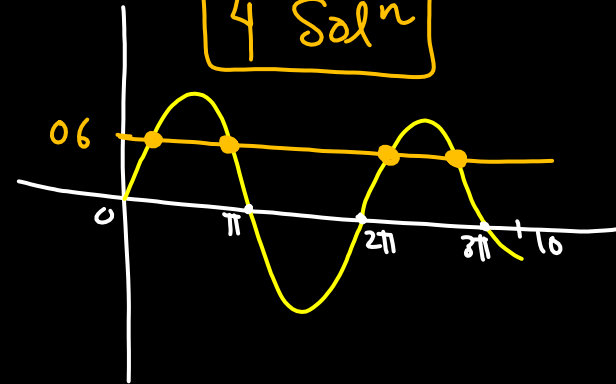
$$\frac{-1 + \sqrt{5}}{2} \approx 0.6$$

$$\frac{-1 - \sqrt{5}}{2} \approx -1.6$$

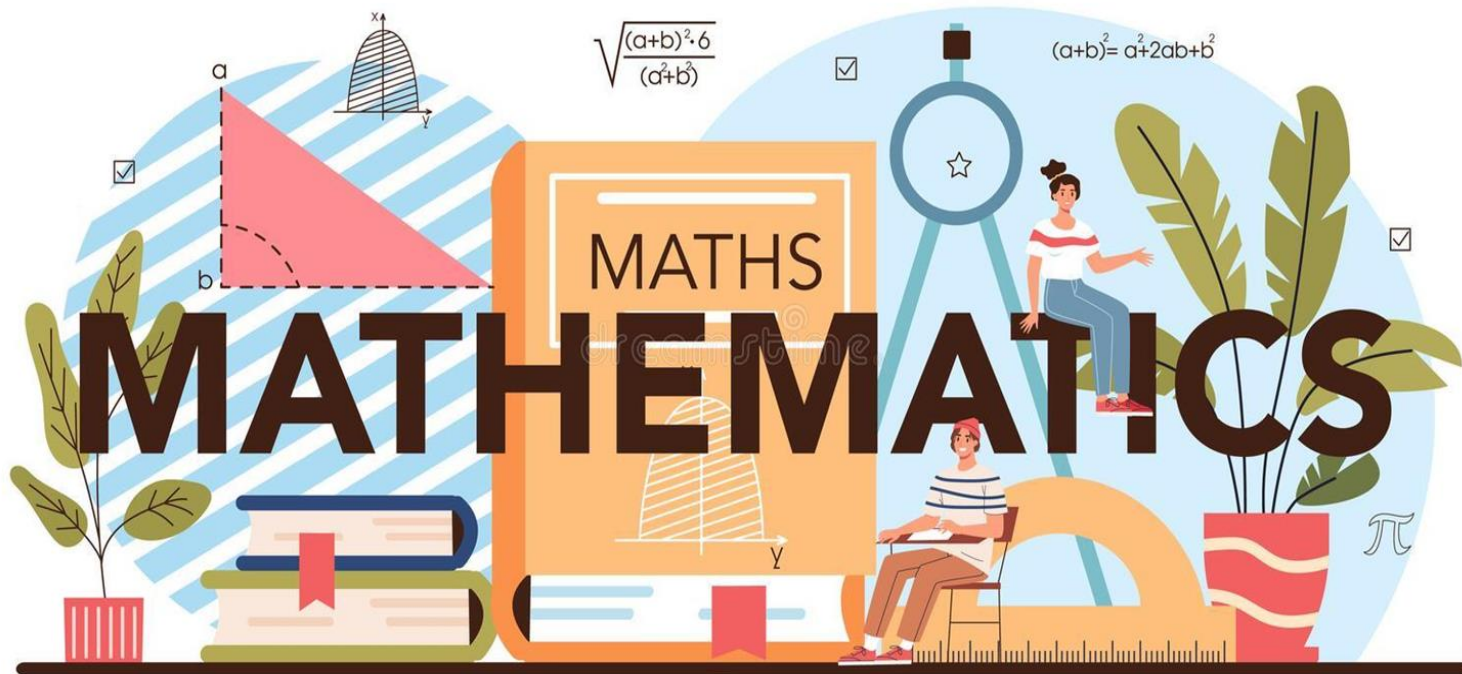


$$\sin x = 0.6 \quad x \in (0, 10)$$

4 Soln



# PYQs



Q

The number of values of  $\theta$  in the interval,  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  such

that  $\theta \neq \frac{n\pi}{5}$  for  $n = 0, \pm 1, \pm 2$  and  $\tan \theta = \cot 5\theta$  as well as

$\sin 2\theta = \cos 4\theta$  is

3

[JEE Adv. 2010]

①

$$\tan \theta = \cot 5\theta$$

$$\tan \theta = \tan\left(\frac{\pi}{2} - 5\theta\right)$$

$$\theta = n\pi + \frac{\pi}{2} - 5\theta$$

$$6\theta = \frac{(2n+1)\pi}{2}$$

$$\theta = \frac{(2n+1)\pi}{12}$$

$$\theta = \frac{(2n+1)\pi}{12}$$

$$\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\theta \in \left(-\frac{6\pi}{12}, \frac{6\pi}{12}\right)$$

$$n=0 \quad \frac{\pi}{12} \quad \checkmark$$

$$n=1 \quad \frac{3\pi}{12} \quad \checkmark$$

$$n=2 \quad \frac{5\pi}{12} \quad \checkmark$$

$$n=-1 \quad -\frac{\pi}{12} \quad \checkmark$$

$$n=-2 \quad -\frac{3\pi}{12} \quad \checkmark$$

$$n=-3 \quad -\frac{5\pi}{12} \quad \checkmark$$



$$\sin 2\theta = \cos 4\theta$$

$$\cos\left(\frac{\pi}{2} - 2\theta\right) = \cos 4\theta$$

$$\frac{\pi}{2} - 2\theta = 2n\pi \pm 4\theta$$

$$\frac{\pi}{2} - 2\theta = 2n\pi + 4\theta$$

$$\frac{\pi}{2} - 2n\pi = 6\theta$$

$$\therefore \boxed{\frac{(1-4n)\pi}{12} = 0}$$

$$\frac{\pi}{2} - 2\theta = 2n\pi - 4\theta$$

$$2\theta = 2n\pi - \frac{\pi}{2}$$

$$\boxed{\theta = \frac{(4n-1)\pi}{4}}$$

$$\theta \in \left( \right) \left\{ \frac{\pi}{12}, \frac{5\pi}{12}, -\frac{3\pi}{12} \right\}$$

$$n=0 \quad \frac{\pi}{12} \checkmark$$

$$n=1 \quad -\frac{3\pi}{12} \checkmark$$

$$n=2 \quad -\frac{7\pi}{12} \times$$

$$n=-1 \quad \frac{5\pi}{12} \checkmark$$

$$n=-2 \quad \frac{9\pi}{12} \times$$

$$\theta \in \left( -\frac{2\pi}{4}, \frac{2\pi}{4} \right)$$

$$n=0 \quad -\frac{\pi}{4} \checkmark$$

$$n=1 \quad \frac{3\pi}{4} \times$$

⋮

$$n=-1 \quad -\frac{5\pi}{4} \times$$



Q

Let  $S = \left\{ \underline{x \in (-\pi, \pi)} : \underline{x \neq 0, \pm \frac{\pi}{2}} \right\}$ . The sum of all distinct

solutions of the equation  $\sqrt{3} \sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$  in the set  $S$  is equal to

(a)  $-\frac{7\pi}{9}$

(b)  $-\frac{2\pi}{9}$

~~(c)~~ 0    (c)

(d)  $\frac{5\pi}{9}$

[JEE Adv. 2016]

$$\sqrt{3} \sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$$

$$\frac{\sqrt{3}}{\cos x} + \frac{1}{\sin x} + 2 \left( \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} \right) = 0$$

$$\frac{\sqrt{3} \sin x + \cos x}{\sin x \cos x} + \frac{2(-\cos 2x)}{\sin x \cos x} = 0$$

$$\sin \frac{\pi}{3} \cos \frac{\pi}{3} + \frac{\sqrt{3}}{2} \sin x + \frac{\cos x}{2} = \cos 2x$$

$$\sin \frac{\pi}{3} \sin x + \cos \frac{\pi}{3} \cos x = \cos 2x$$

$$\cos \left( x - \frac{\pi}{3} \right) = \cos 2x$$

$$x - \frac{\pi}{3} = 2n\pi + 2x$$

$$-\frac{\pi}{3} - 2n\pi = x$$

$$\boxed{-\frac{(6n+1)\pi}{3} = x}$$

$$x \in (-\pi, \pi) = \left( -\frac{9\pi}{9}, \frac{9\pi}{9} \right)$$

$$\boxed{n=0 \quad -\frac{\pi}{3}} \checkmark$$

$$n=1 \quad -\frac{7\pi}{3} \times$$

$$n=-1 \quad \frac{5\pi}{3} \times$$

$$n=0 \quad \frac{\pi}{9} \checkmark$$

$$n=1 \quad \frac{7\pi}{9} \checkmark$$

$$n=2 \quad \frac{13\pi}{9} \times$$

$$n=-1 \quad -\frac{5\pi}{9} \checkmark$$

$$n=-2 \quad -\frac{11\pi}{9} \times$$

$$-\frac{\pi}{3} + \frac{\pi}{9} + \frac{7\pi}{9} = \frac{5\pi}{9}$$

$$x - \frac{\pi}{3} = 2n\pi - 2x$$

$$3x = 2n\pi + \frac{\pi}{3}$$

$$\boxed{x = \frac{(6n+1)\pi}{9}}$$



# Join with us in Telegram

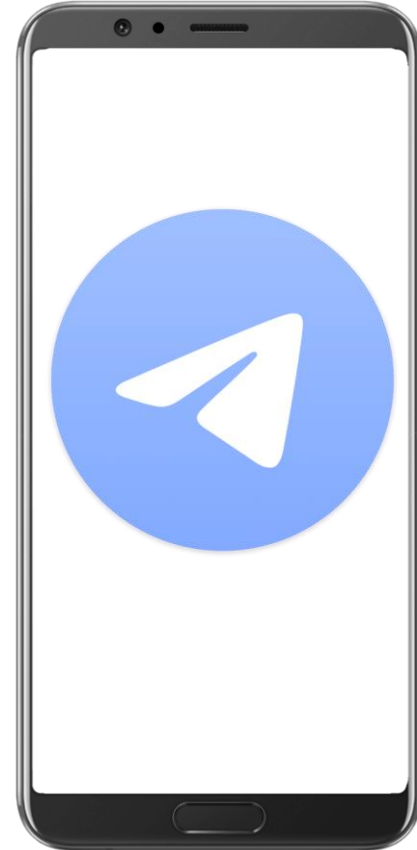
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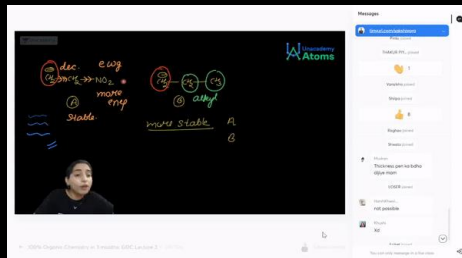
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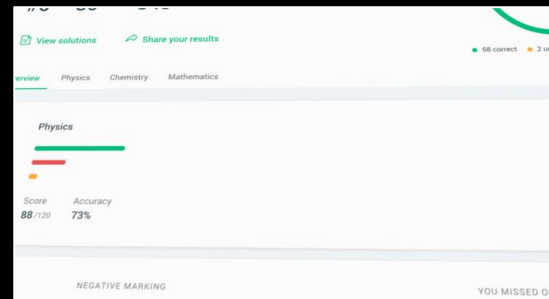
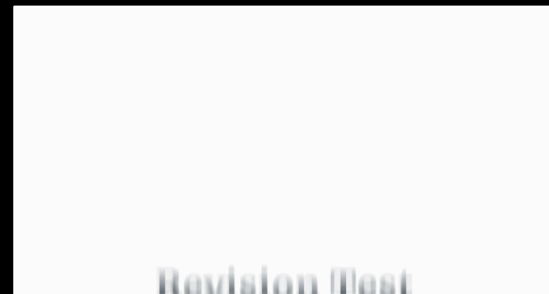
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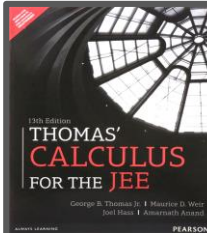
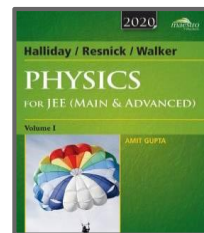
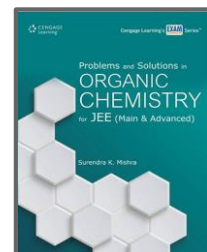
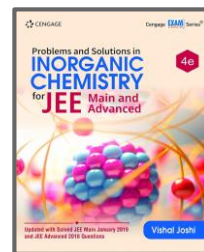
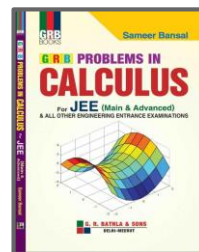
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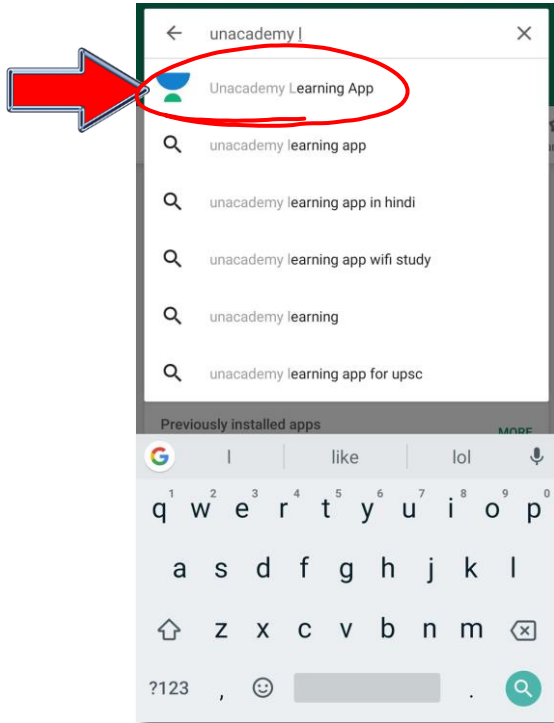
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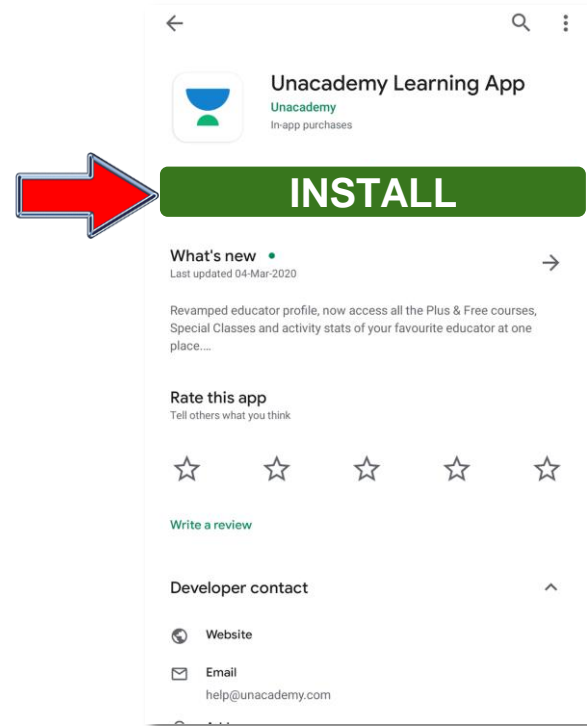
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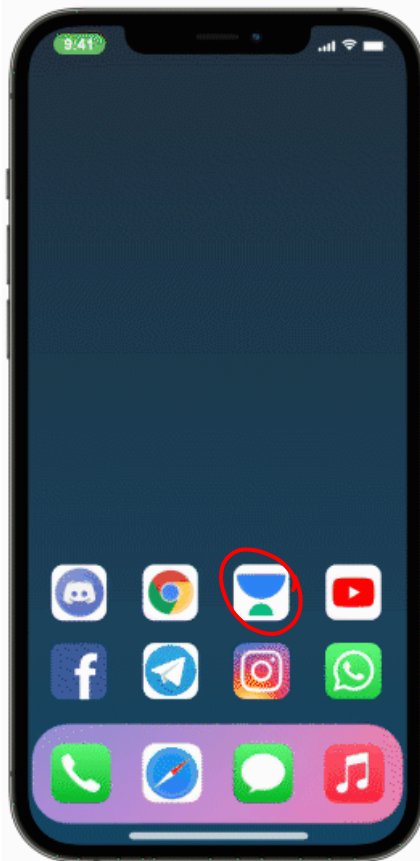
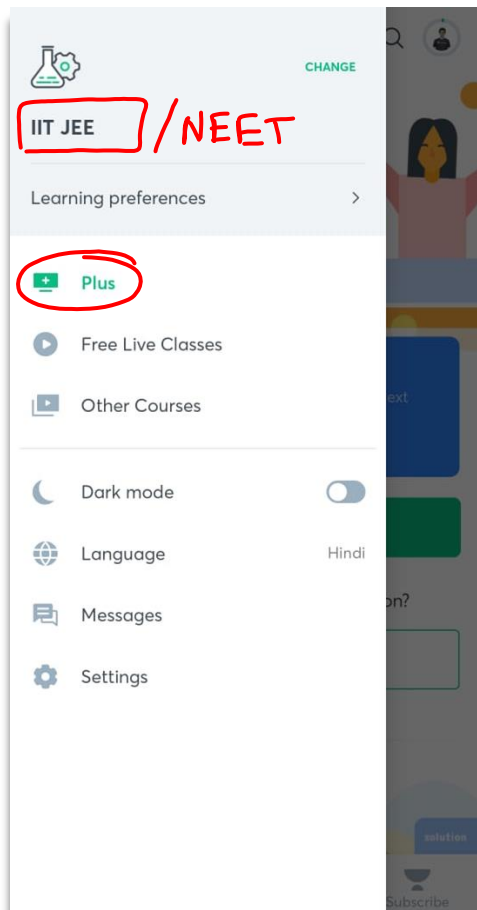
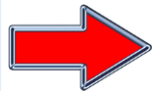


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
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