CLASS X : CHAPTER - 3
PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

NCERT NICHOOD

- An equation of the form ax + by + c = 0, where a, b and c are real numbers ($a \neq 0, b \neq 0$), is called a linear equation in two variables x and y.
- The numbers a and b are called the coefficients of the equation ax + by + c = 0 and the number c is called the constant of the equation ax + by + c = 0.

Two linear equations in the same two variables are called a pair of linear equations in two variables. The most general form of a pair of linear equations is

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

where $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers, such that $a_1^2 + b_1^2 \neq 0,\ a_2^2 + b_2^2 \neq 0.$

CONSISTENT SYSTEM

A system of simultaneous linear equations is said to be consistent, if it has at least one solution.

INCONSISTENT SYSTEM

A system of simultaneous linear equations is said to be inconsistent, if it has no solution.

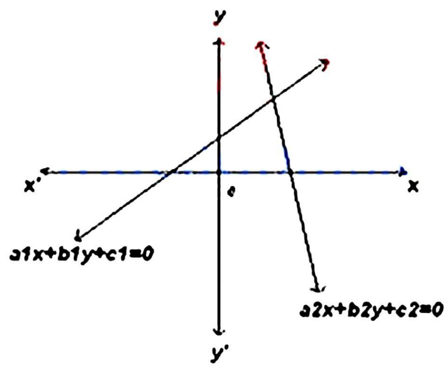
METHOD TO SOLVE A PAIR OF LINEAR EQUATION OF TWO VARIABLES

A pair of linear equations in two variables can be represented, and solved, by the:
(i) graphical method (ii) algebraic method

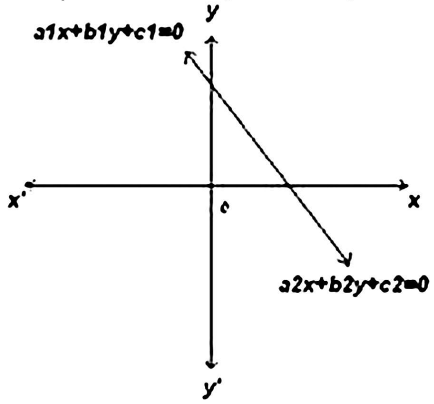
GRAPHICAL METHOD OF SOLUTION OF A PAIR OF LINEAR EQUATIONS

The graph of a pair of linear equations in two variables is represented by two lines.

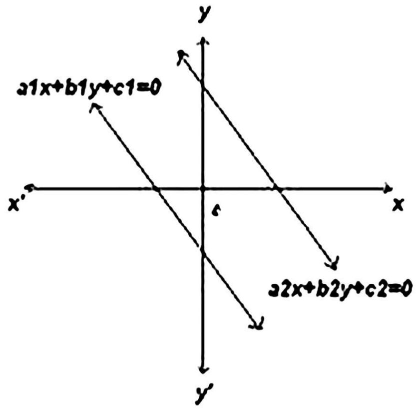
1. If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is **consistent**.



2. If the lines coincide, then there are infinitely many solutions — each point on the line being a solution. In this case, the pair of equations is **dependent (consistent)**.



3. If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is **inconsistent**.



Algebraic interpretation of pair of linear equations in two variables

The pair of linear equations represented by these lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

1. If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then the pair of linear equations has exactly one solution.
2. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then the pair of linear equations has infinitely many solutions.
3. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then the pair of linear equations has no solution.

S. No.	Pair of lines	Compare the ratios	Graphical representation	Algebraic interpretation
1	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Unique solution (Exactly one solution)
2	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions
3	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

ALGEBRAIC METHODS OF SOLVING A PAIR OF LINEAR EQUATIONS

Substitution Method

Following are the steps to solve the pair of linear equations by substitution method:

$a_1x + b_1y + c_1 = 0 \cdots$ (i) and

$a_2x + b_2y + c_2 = 0 \cdots$ (ii)

Step 1: We pick either of the equations and write one variable in terms of the other

$$y = -\frac{a_1}{b_1}x - \frac{c_1}{b_1} \cdots \text{(iii)}$$

Step 2: Substitute the value of x in equation (i) from equation (iii) obtained in step 1.

Step 3: Substituting this value of y in equation (iii) obtained in step 1, we get the values of x and y.

Elimination Method

Following are the steps to solve the pair of linear equations by elimination method:

Step 1: First multiply both the equations by some suitable non-zero constants to make the coefficients of one variable (either x or y) numerically equal.

Step 2: Then add or subtract one equation from the other so that one variable gets eliminated.

- ❖ If you get an equation in one variable, go to Step 3.
- ❖ If in Step 2, we obtain a true statement involving no variable, then the original pair of equations has infinitely many solutions.

- If in Step 2, we obtain a false statement involving no variable, then the original pair of equations has no solution, i.e., it is inconsistent.

Step 3: Solve the equation in one variable (x or y) so obtained to get its value.
Step 4: Substitute this value of x (or y) in either of the original equations to get the value of the other variable.

Cross - Multiplication Method

Let the pair of linear equations be:

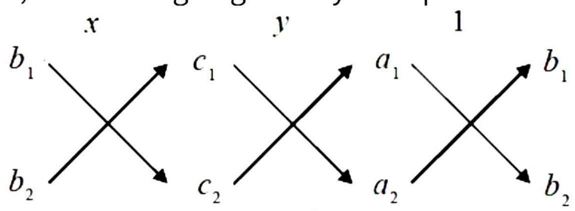
$a_1x + b_1y + c_1 = 0 \cdots (1)$ and

$a_2x + b_2y + c_2 = 0 \cdots (2)$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1} \quad \cdots\cdots\cdots\cdots. (3)$$

$$\Rightarrow x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad and \quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

In remembering the above result, the following diagram may be helpful :



The arrows between the two numbers indicate that they are to be multiplied and the second product is to be subtracted from the first.

For solving a pair of linear equations by this method, we will follow the following steps :
Step 1 : Write the given equations in the form (1) and (2).
Step 2 : Taking the help of the diagram above, write Equations as given in (3).
Step 3 : Find ***x*** and ***y***, provided $a_1b_2 - a_2b_1 \neq 0$

Step 2 above gives you an indication of why this method is called the **cross-multiplication method**.