CLASS X : CHAPTER - 2
POLYNOMIALS

NCERT NICHOOD

An algebraic expression of the form $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots\cdots\cdots . a_nx^n$, where $a \neq 0$, is called a polynomial in variable x of degree n.
Here, $a_0, a_1, a_2, a_3, \cdots\cdots, a_n$ are real numbers and each power of x is a non-negative integer.
e.g. $3x^2 - 5x + 2$ is a polynomial of degree 2.
$3\sqrt{x} + 2$ is not a polynomial.

- If $p(x)$ is a polynomial in x, the highest power of x in $p(x)$ is called **the degree of the polynomial $p(x)$**. For example, $4x + 2$ is a polynomial in the variable x of degree 1, $2y^2 - 3y + 4$ is a polynomial in the variable y of degree 2,

  - A polynomial of degree 0 is called a constant polynomial.
  - A polynomial $p(x) = ax + b$ of degree 1 is called a linear polynomial.
  - A polynomial $p(x) = ax^2 + bx + c$ of degree 2 is called a quadratic polynomial.
  - A polynomial $p(x) = ax^3 + bx^2 + cx + d$ of degree 3 is called a cubic polynomial.
  - A polynomial $p(x) = ax^4 + bx^3 + cx^2 + dx + e$ of degree 4 is called a bi-quadratic polynomial.

VALUE OF A POLYNOMIAL AT A GIVEN POINT x = k

If $p(x)$ is a polynomial in x, and if k is any real number, then the value obtained by replacing x by k in $p(x)$, is called **the value of $p(x)$ at $x = k$**, and is denoted by $p(k)$.

ZERO OF A POLYNOMIAL

A real number k is said to be a **zero of a polynomial $p(x)$**, if $p(k) = 0$.

- Geometrically, the zeroes of a polynomial $p(x)$ are precisely the x-coordinates of the points, where the graph of $y = p(x)$ intersects the x -axis.
- A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most 3 zeroes.
- In general, a polynomial of degree 'n' has at the most 'n' zeroes.

RELATIONSHIP BETWEEN ZEROES & COEFFICIENTS OF POLYNOMIALS

Type of Polynomial	General form	No. of zeroes	Relationship between zeroes and coefficients
Linear	$ax + b, a \neq 0$	1	$k = -\frac{b}{a}$, i.e. $k = -\frac{\text{Constant term}}{\text{Coefficient of x}}$
Quadratic	$ax^2 + bx + c, a \neq 0$	2	Sum of zeroes $(\alpha + \beta) = -\frac{\text{Coefficient of x}}{\text{Coefficient of x}^2} = -\frac{b}{a}$ Product of zeroes $(\alpha\beta) = \frac{\text{Constant term}}{\text{Coefficient of x}^2} = \frac{c}{a}$
Cubic	$ax^3 + bx^2 + cx + d$, $a \neq 0$	3	Sum of zeroes $(\alpha + \beta + \gamma) = -\frac{\text{Coefficient of x}^2}{\text{Coefficient of x}^3} = -\frac{b}{a}$ Product of sum of zeroes taken two at a time $(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{\text{Coefficient of x}}{\text{Coefficient of x}^3} = \frac{c}{a}$ Product of zeroes $(\alpha\beta\gamma) = -\frac{\text{Constant term}}{\text{Coefficient of x}^3} = -\frac{d}{a}$

- ❖ A quadratic polynomial whose zeroes are α and β is given by $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$
  i.e. x^2 – (Sum of zeroes)x + (Product of zeroes)

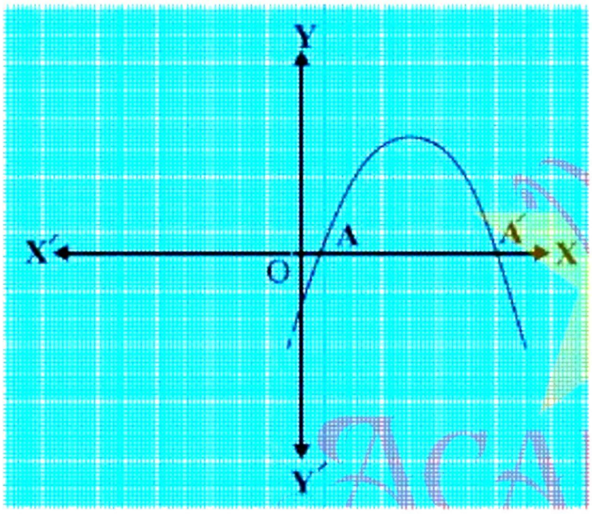
- ❖ A cubic polynomial whose zeroes are α, β ***and*** γ is given by
  $p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

The zeroes of a quadratic polynomial $ax^2 + bx + c$, $a \neq 0$, are precisely the x-coordinates of the points where the parabola representing $y = ax^2 + bx + c$ intersects the x-axis.

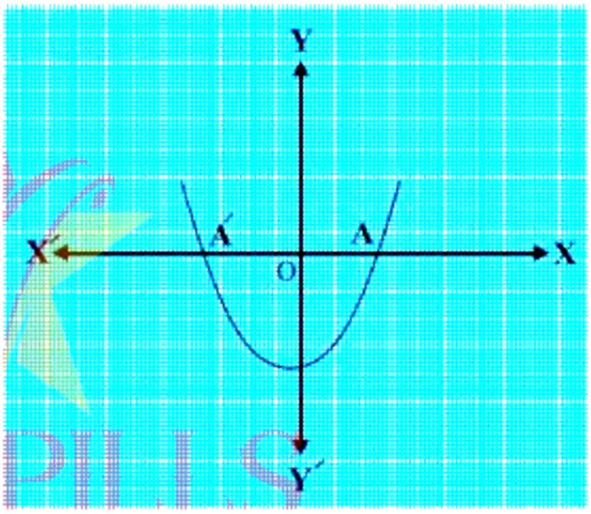
In fact, for any quadratic polynomial $ax^2 + bx + c$, $a \neq 0$, the graph of the corresponding equation $y = ax^2 + bx + c$ has one of the two shapes either open upwards like $\cup$ or open downwards like $\cap$ depending on whether $a > 0$ or $a < 0$. (These curves are called **parabolas**.)

The following three cases can be happen about the graph of quadratic polynomial $ax^2 + bx + c$:

Case (i) : Here, the graph cuts x-axis at two distinct points A and A'. The x-coordinates of A and A' are the **two zeroes** of the quadratic polynomial $ax^2 + bx + c$ in this case

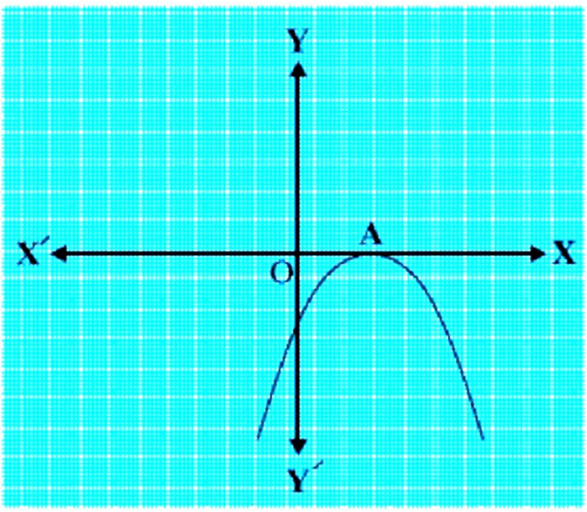


(i)
a > 0

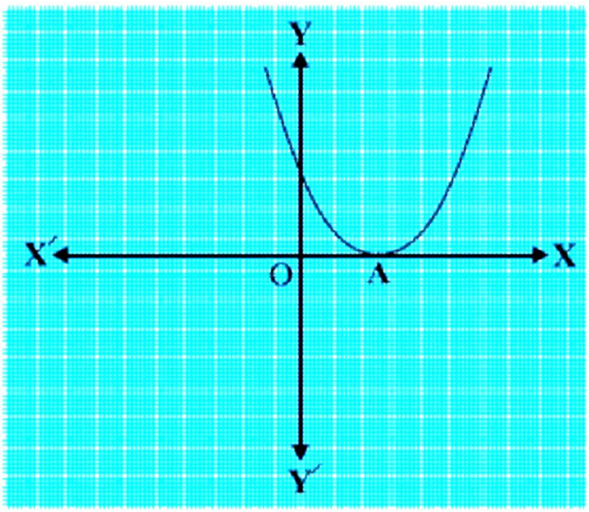


(ii)
a < 0

Case (ii) : Here, the graph cuts the x-axis at exactly one point, i.e., at two coincident points. So, the two points A and A′ of Case (i) coincide here to become one point A. The x-coordinate of A is the **only zero** for the quadratic polynomial $ax^2 + bx + c$ in this case.

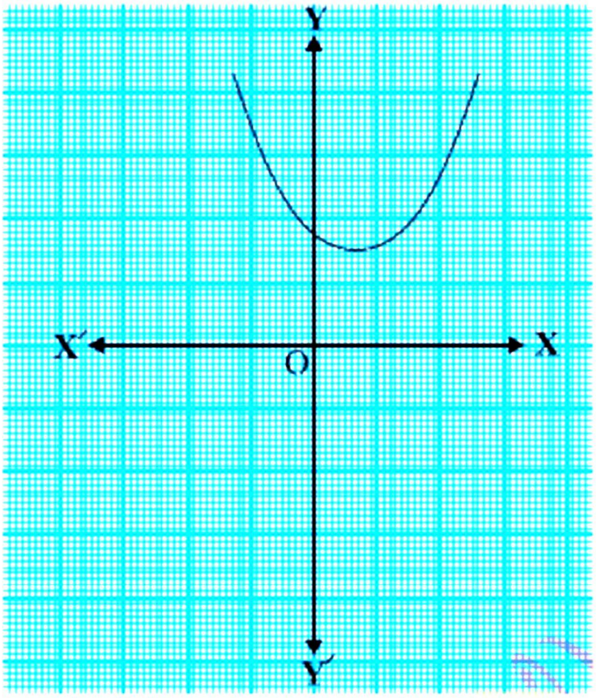


(i)
a > 0



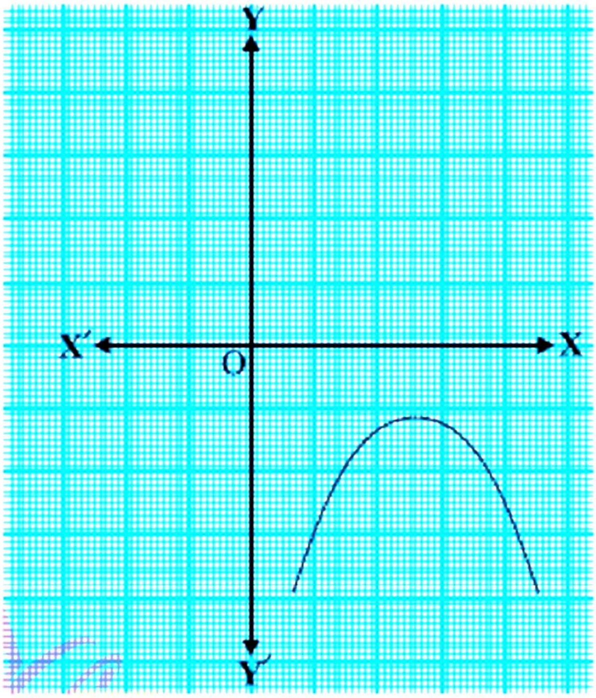
(ii)
a < 0

Case (iii) : Here, the graph is either completely above the *x*-axis or completely below the *x*-axis. So, it does not cut the *x*-axis at any point. So, the quadratic polynomial $ax^2 + bx + c$ has **no zero** in this case.



(i)

a > 0



(ii)

a < 0

DIVISION ALGORITHM FOR POLYNOMIALS

If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that $p(x) = g(x) \times q(x) + r(x)$,
where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.

- If r(x) = 0, then g(x) is a factor of p(x).
- Dividend = Divisor × Quotient + Remainder