CLASS X : CHAPTER - 4
QUADRATIC EQUATIONS

NCERT NICHOOD

POLYNOMIALS

An algebraic expression of the form $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots\cdots\cdots a_nx^n$, where $a \neq 0$, is called a polynomial in variable x of degree n.
Here, $a_0, a_1, a_2, a_3, \cdots\cdots\cdots, a_n$ are real numbers and each power of x is a non-negative integer.
e.g. $3x^2 - 5x + 2$ is a polynomial of degree 2.
$3\sqrt{x} + 2$ is not a polynomial.

- If $p(x)$ is a polynomial in x, the highest power of x in $p(x)$ is called **the degree of the polynomial** $p(x)$. For example, $4x + 2$ is a polynomial in the variable x of degree 1, $2y^2 - 3y + 4$ is a polynomial in the variable y of degree 2,

    - A polynomial of degree 0 is called a constant polynomial.
    - A polynomial $p(x) = ax + b$ of degree 1 is called a linear polynomial.
    - A polynomial $p(x) = ax^2 + bx + c$ of degree 2 is called a quadratic polynomial.
    - A polynomial $p(x) = ax^3 + bx^2 + cx + d$ of degree 3 is called a cubic polynomial.
    - A polynomial $p(x) = ax^4 + bx^3 + cx^2 + dx + e$ of degree 4 is called a bi-quadratic polynomial.

QUADRATIC EQUATION

A polynomial $p(x) = ax^2 + bx + c$ of degree 2 is called a quadratic polynomial, then $p(x) = 0$ is known as quadratic equation.
e.g. $2x^2 - 3x + 2 = 0$, $x^2 + 5x + 6 = 0$ are quadratic equations.

METHODS TO FIND THE SOLUTION OF QUADRATIC EQUATIONS

Three methods to find the solution of quadratic equation:

1. Factorisation method
2. Method of completing the square
3. Quadratic formula method

FACTORISATION METHOD

Steps to find the solution of given quadratic equation by factorisation

- Firstly, write the given quadratic equation in standard form $ax^2 + bx + c = 0$.
- Find two numbers α and β such that sum of α and β is equal to b and product of α and β is equal to ac.
- Write the middle term bx as $\alpha x + \beta x$ and factorise it by splitting the middle term and let factors are $(x + p)$ and $(x + q)$ i.e. $ax^2 + bx + c = 0 \Rightarrow (x + p)(x + q) = 0$
- Now equate reach factor to zero and find the values of x.
- These values of x are the required roots/solutions of the given quadratic equation.

METHOD OF COMPLETING THE SQUARE

Steps to find the solution of given quadratic equation by Method of completing the square:

- Firstly, write the given quadratic equation in standard form $ax^2 + bx + c = 0$.
- Make coefficient of x^2 unity by dividing all by a then we get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

- Shift the constant on RHS and add square of half of the coefficient of x i.e. $\left(\frac{b}{2a}\right)^2$ on both sides.

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \Rightarrow x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

- Write LHS as the perfect square of a binomial expression and simplify RHS.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

- Take square root on both sides

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

- Find the value of x by shifting the constant term on RHS i.e. $x = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} - \frac{b}{2a}$

QUADRATIC FORMULA METHOD

Steps to find the solution of given quadratic equation by quadratic formula method:

- Firstly, write the given quadratic equation in standard form $ax^2 + bx + c = 0$.
- Write the values of a, b and c by comparing the given equation with standard form.
- Find discriminant $D = b^2 - 4ac$. If value of D is negative, then is no real solution i.e. solution does not exist. If value of $D \geq 0$, then solution exists follow the next step.
- Put the value of a, b and D in quadratic formula $x = \frac{-b \pm \sqrt{D}}{2a}$ and get the required roots/solutions.

NATURE OF ROOTS

The roots of the quadratic equation $ax^2 + bx + c = 0$ by quadratic formula are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

where $D = b^2 - 4ac$ is called discriminant. The nature of roots depends upon the value of discriminant D. There are three cases –

Case – I

When $D > 0$ i.e. $b^2 - 4ac > 0$, then the quadratic equation has two distinct roots.

i.e. $x = \frac{-b + \sqrt{D}}{2a}$ and $\frac{-b - \sqrt{D}}{2a}$

Case – II

When $D = 0$, then the quadratic equation has two equal real roots.

i.e. $x = \frac{-b}{2a}$ and $\frac{-b}{2a}$

Case – III

When $D < 0$ then there is no real roots exist.