CLASS X : CHAPTER - 1
REAL NUMBERS
NCERT NICHOOD

EUCLID'S DIVISION LEMMA
Given positive integers a and b, there exist unique integers q and r satisfying a = bq + r, where $0 \le r < b$.
Here we call 'a' as dividend, 'b' as divisor, 'q' as quotient and 'r' as remainder.
$\therefore$ Dividend = (Divisor x Quotient) + Remainder
If in Euclid's lemma r = 0 then b would be HCF of 'a' and 'b'.

NATURAL NUMBERS
Counting numbers are called natural numbers i.e. 1, 2, 3, 4, 5, ……………. are natural numbers.

WHOLE NUMBERS
All counting numbers/natural numbers along with 0 are called whole numbers i.e. 0, 1, 2, 3, 4, 5 ……………. are whole numbers.

INTEGERS
All natural numbers, negative of natural numbers and 0, together are called integers. i.e.
………. – 3, – 2, – 1, 0, 1, 2, 3, 4, …………….. are integers.

ALGORITHM
An **algorithm** is a series of well defined steps which gives a procedure for solving a type of problem.

LEMMA
A **lemma** is a proven statement used for proving another statement.

EUCLID'S DIVISION ALGORITHM
Euclid's division algorithm is a technique to compute the Highest Common Factor (HCF) of two given positive integers. Recall that the HCF of two positive integers ***a*** and ***b*** is the largest positive integer ***d*** that divides both ***a*** and ***b***.

To obtain the HCF of two positive integers, say c and d, with c > d, follow the steps below:
Step 1 : Apply Euclid's division lemma, to ***c*** and ***d***. So, we find whole numbers, ***q*** and ***r*** such that $c = dq + r$, $0 \le r < d$.
Step 2 : If $r = 0$, ***d*** is the HCF of ***c*** and ***d***. If $r \ne 0$ apply the division lemma to ***d*** and ***r***.
Step 3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.
This algorithm works because HCF (***c***, ***d***) = HCF (***d***, ***r***) where the symbol HCF (***c***, ***d***) denotes the HCF of ***c*** and ***d***, etc.

The Fundamental Theorem of Arithmetic
Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

The prime factorisation of a natural number is unique, except for the order of its factors.

- HCF is the highest common factor also known as GCD i.e. greatest common divisor.
- LCM of two numbers is their least common multiple.
- Property of HCF and LCM of two positive integers 'a' and 'b':

- $HCF(a,b) \times LCM(a,b) = a \times b$
- $LCM(a,b) = \frac{a \times b}{HCF(a,b)}$
- $HCF(a,b) = \frac{a \times b}{LCM(a,b)}$

PRIME FACTORISATION METHOD TO FIND HCF AND LCM

HCF(a, b) = Product of the smallest power of each common prime factor in the numbers.
LCM (a, b) = Product of the greatest power of each prime factor, involved in the numbers.

RATIONAL NUMBERS

The number in the form of $\frac{p}{q}$ where 'p' and 'q' are integers and $q \neq 0$, e.g. $\frac{2}{3}, \frac{3}{5}, \frac{5}{7}, \ldots\ldots$

Every rational number can be expressed in decimal form and the decimal form will be either terminating or non-terminating repeating. e.g. $\frac{5}{2} = 2.5$ (Terminating), $\frac{2}{3} = 0.66666....$ *or* $0.\bar{6}$ (Non-terminating repeating).

IRRATIONAL NUMBERS

The numbers which are not rational are called irrational numbers. e.g. $\sqrt{2}, \sqrt{3}, \sqrt{5}, etc$.

- ***Let p be a prime number. If p divides a2, then p divides a, where a is a positive integer.***
- If p is a positive integer which is not a perfect square, then $\sqrt{m}$ is an irrational, e.g. $\sqrt{2}, \sqrt{5}, \sqrt{6}, \sqrt{8}, ...etc$.
- If p is prime, then $\sqrt{p}$ is also an irrational.

RATIONAL NUMBERS AND THEIR DECIMAL EXPANSIONS

- ***Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form $\frac{p}{q}$ where p and q are coprime, and the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers.***

- ***Let x = $\frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.***
- ***Let x = $\frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form $2^n 5^m$, where n, m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating (recurring).***

- ***The decimal form of irrational numbers is non-terminating and non-repeating.***
- ***Those decimals which are non-terminating and non-repeating will be irrational numbers. e.g. 0.20200200020002… … . is a non-terminating and non-repeating decimal, so it irrational.***