ALTERNATING CURRENT

ALTERNATING CURRENT -

The current whose magnitude changes continuously with time between zero and a maximum value and whose direction reverses periodically.

$$\boxed{I = I_0 \sin \omega t} \Rightarrow I = I_0 \sin 2\pi \nu t$$

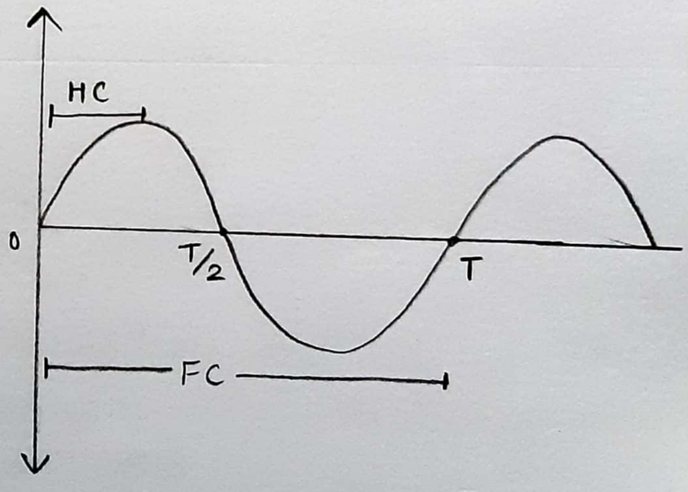
where, ω = angular frequency

I_0 = peak value of AC

AVERAGE OR MEAN VALUE OF AC

$I = I_0 \sin \omega t$

$$I_{av.} = \int_0^{T/2} \frac{I dt}{T/2 - 0}$$



$$I_{av.} = \int_0^{T/2} \frac{I_0 \sin \omega t \, dt}{T/2}$$

$$I_{av.} = \frac{2}{T} I_0 \int_0^{T/2} \sin \omega t \, dt$$

$$I_{av.} = \frac{2}{T} I_0 \left[-\frac{\cos \omega t}{\omega} \right]_0^{T/2}$$

$$I_{av.} = \frac{-2}{\omega T} I_0 \left[\cos \omega t \right]_0^{T/2}$$

$$I_{av.} = -\frac{2 I_0 T}{2\pi T} \left[\cos \frac{2\pi}{\cancel{T}} \times \frac{\cancel{T}}{2} - 1 \right]$$

$$I_{av.} = \frac{-I_0}{\pi} [-2]$$

$$\boxed{I_{av.} = \frac{2 I_0}{\pi}}$$

ROOT MEAN SQUARE VALUE OF CURRENT -

$P = I^2 R$

$I = I_0 \sin \omega t$

$P = (I_0 \sin \omega t)^2 R$

$P = I_0^2 \sin^2 \omega t \, R$

$\frac{dH}{dt} = I_0^2 R \sin^2 \omega t$

$dH = I_0^2 R \sin^2 \omega t \, dt$

$H = \int_0^T I_0^2 R \sin^2 \omega t \, dt$

$H = I_0^2 R \int_0^T \sin^2 \omega t \, dt$

$\sin^2\theta = \frac{1 - \cos^2\theta}{2}$

$H = \frac{I_0^2 R}{2} \int_0^T (1 - \cos^2 \omega t) \, dt$

$H = \frac{I_0^2 R}{2} \left[\int_0^T dt - \int_0^T \cancel{\cos 2 \omega t \, dt} \right]$

$H = \frac{I_0^2 RT}{2}$

$AC \rightarrow H_{ac.} = \frac{I_0^2 RT}{2}$ $\qquad$ $DC \rightarrow H_{ac.} = I_{RMS}^2 RT$

$\frac{I_0^2 RT}{2} = I_{RMS}^2 RT$

$\boxed{I_{RMS} = \frac{I_0}{\sqrt{2}}}$ $\qquad$ $\boxed{V_{RMS} = \frac{V_0}{\sqrt{2}}}$

INDUCTIVE REACTANCE (X_L) -

The effective resistance or opposition offered by the inductor to the flow of current is inductive reactance.

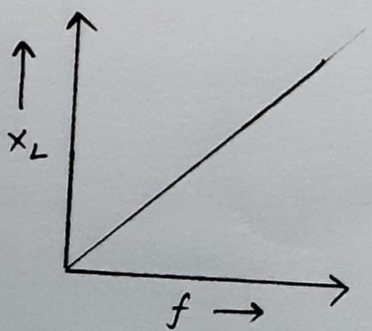
$X_L = \omega L = 2 \pi f L$

$\boxed{X_L = (2 \pi L) f}$

$X_L \propto f$

L = self inductance

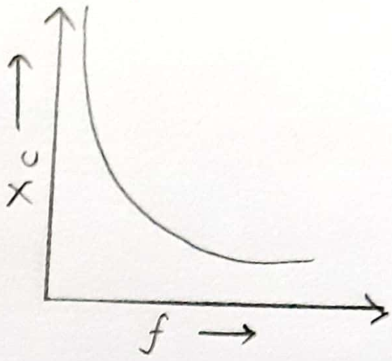
$[\because 2 \pi L = \text{constant}]$



CAPACITIVE REACTANCE (X_C)

The opposing nature of capacitor to the flow of alternating current is called capacitive reactance.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$



$$X_C = \left(\frac{1}{2\pi C}\right) \cdot \frac{1}{f} \Rightarrow X_C \propto \frac{1}{f}$$

C = capacitance of AC

[$\because$ $1/2\pi C$ = constant]

WATTLESS CURRENT–

The current in purely inductive or capacitive AC circuit when average power consumption in AC circuit is zero is wattless current.

PHASOR DIAGRAM–

The representation of AC current and voltage by rotating vectors is called phasor diagram.

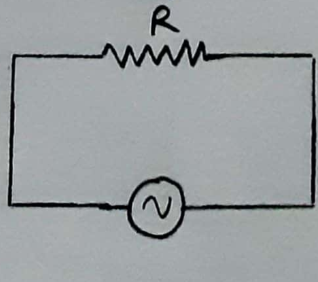
TYPES OF AC CIRCUITS–

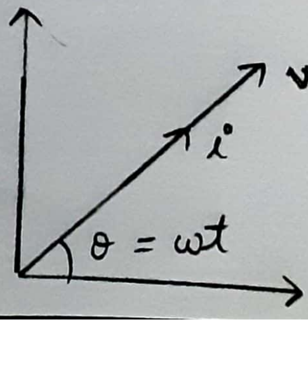
1. AC THROUGH RESISTOR

When AC is applied to resistor, I and V are in phase or there is no phase difference between I and V

$$V = V_0 \sin \omega t$$

$$I = \frac{V}{R}$$

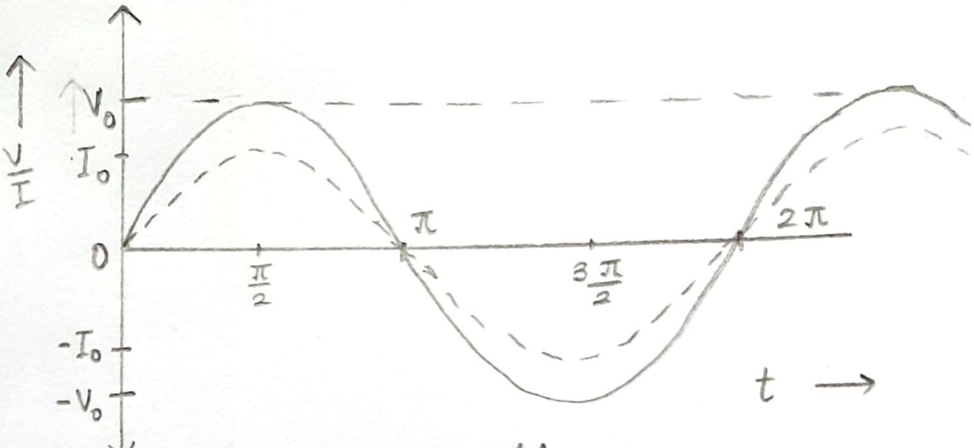




$$I = \frac{V_0 \sin \omega t}{R}$$

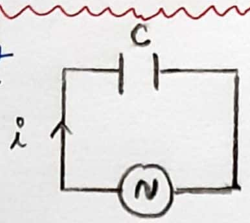
$$\boxed{I = I_0 \sin \omega t}$$

$$\left[I_0 = \frac{V_0}{R} \right]$$



V-I graph →

2. AC THROUGH CAPACITOR



$$V = V_0 \sin \omega t$$

$$Q = CV$$

$$I = \frac{dq}{dt}$$

$$I = \frac{d}{dt} CV$$

$$I = C \frac{dV}{dt}$$

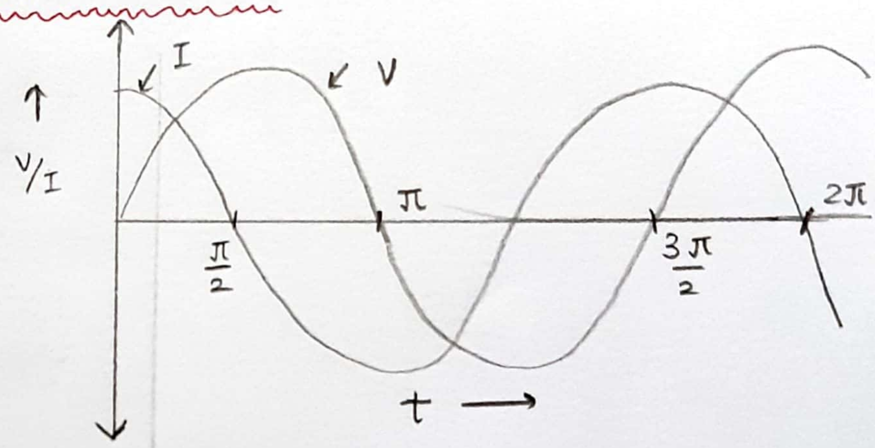
$$I = C \frac{d}{dt} (V_0 \sin \omega t)$$

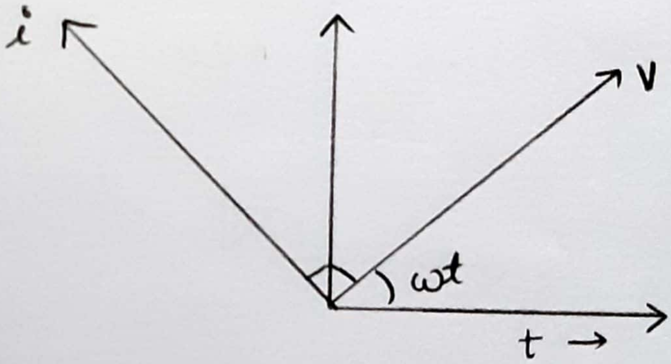
$$I = V_0 C \omega \cos \omega t$$

$$I = \frac{V_0}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\boxed{I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)}$$

$$I_0 = \frac{V}{1/\omega C}$$

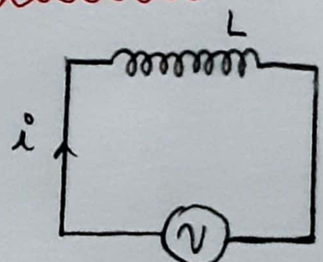




3. AC THROUGH INDUCTOR

$$V = L \frac{di}{dt}$$

$$V_0 \sin \omega t = L \frac{di}{dt}$$



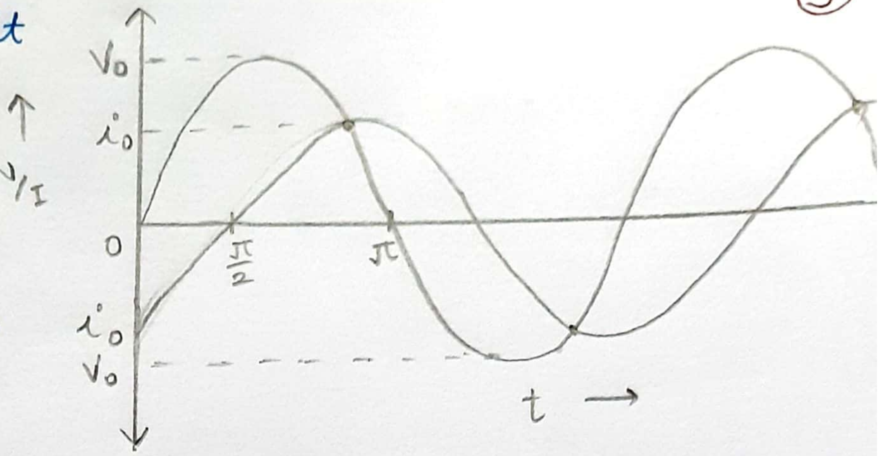
$$\int di = \int \frac{V_0}{L} \sin \omega t \, dt$$

$$i = \frac{V_0}{\omega L} [-\cos \omega t]$$

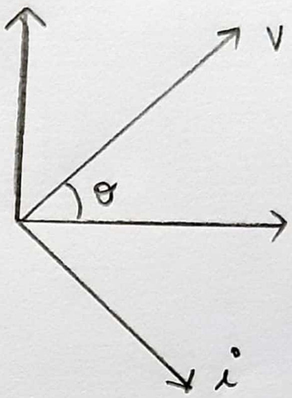
$$i = \frac{-V_0}{\omega L} \cos \omega t$$

$$I = \frac{-V_0}{\omega L} [\sin 90 - \omega t]$$

$$I = \frac{V_0}{\omega L} \left[\sin \omega t - \frac{\pi}{2}\right]$$

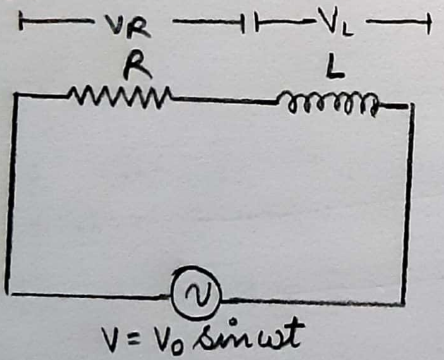


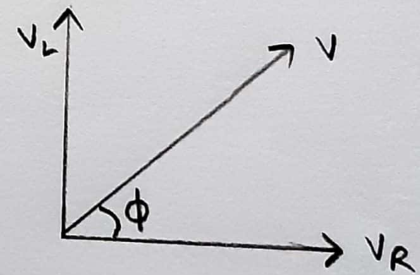
$$\boxed{I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)}$$



$$I_0 = \frac{V_0}{\omega L} \quad ; \quad I_0 = \frac{V_0}{X_L}$$

L-R series AC circuit





Then,

(i) Impedance, $Z = \sqrt{R^2 + X_L^2}$

$= \sqrt{R^2 + \omega^2 L^2} \quad [\because X_L = \omega L]$

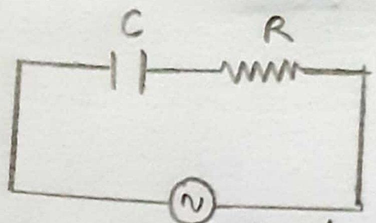
$= V_{rms} / I_{rms}$

(ii) For the phase angle, $\tan \phi = \frac{X_L}{R} = \frac{\omega L}{R}$

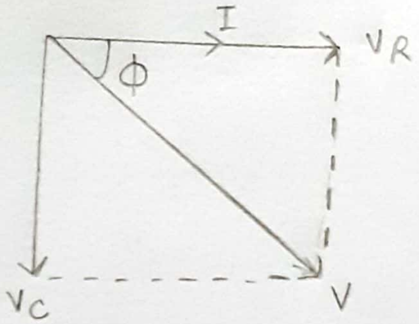
(iii) If $V = V_0 \sin \omega t$, then $I = I_0 \sin(\omega t - \phi)$

* Voltage leads current by phase ϕ

R-C series AC circuit



$V = V_0 \sin \omega t$



Then,

(i) Impedence, $Z = \frac{V_{rms}}{I_{rms}} = \sqrt{R^2 + X_C^2}$

$$= \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \quad \left[\because X_C = \frac{1}{\omega C}\right]$$

(ii) For the phase angle, $\tan\phi = \frac{X_C}{R} = \frac{1}{\omega C R}$

(iii) If $V = V_0 \sin \omega t$, then $I = I_0 \sin(\omega t + \phi)$

(iv) Power factor, $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_C^2}}$

* Current ahead of voltage by ϕ

L-C series AC circuit

(i) Impedence, $Z = \frac{V_{rms}}{I_{rms}}$

(ii) Applied voltage = $V_L - V_C$

(iii) Phase difference between voltage and current is $\pi/2$

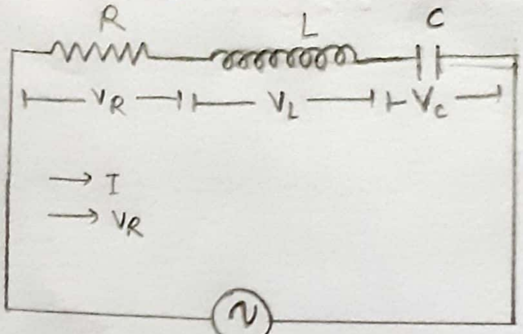
(iv) Power factor, $\cos\phi = 0$

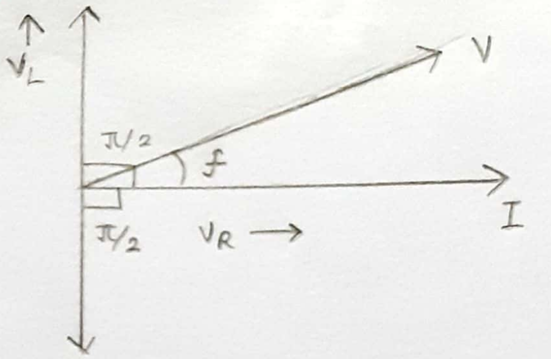
(v) Current, $I = I_0 \sin\left(\omega t \pm \frac{\pi}{2}\right)$

Impedance -

It is the total resistance applied in the path of alternating current. It is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$

L-C-R series AC Circuit





Then,

(i) Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \dfrac{V_{rms}}{I_{rms}}$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

(ii) If net reactance is inductive, circuit behaves as L-R circuit.

(iii) If net reactance is capacitive, circuit behaves as C-R circuit.

RESONANT LCR CIRCUIT

(i) $X_L = X_C$

(ii) Impedance, $Z = Z_{min.} = R$ i.e. circuit behaves as resistive circuit.

(iii) The phase difference between V and I is 0°.

(iv) Resonant angular frequency, $\omega_r = \dfrac{1}{\sqrt{LC}}$

(v) Average power consumption P_{av} becomes maximum

QUALITY FACTOR

The characteristic of a series resonant circuit is determinant by Q-factor. It indicates the sharpness of resonance in an L-C-R series AC circuit.

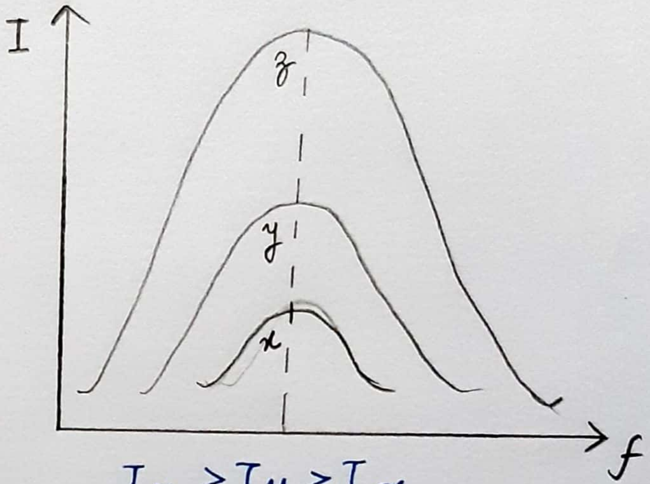
$$Q = \frac{V_L}{V_R} = \frac{V_C}{V_R} = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \quad ; \quad \boxed{Q = \frac{\omega_0}{\omega_2 - \omega_1}}$$

RESONANCE

In series LCR circuit, when phase (ϕ) between current and voltage is zero, the circuit is said to be resonant circuit.

The frequency at which X_C and X_L become equal is called resonant frequency.

$$V_L = I X_L$$
$$I X_L = I X_C$$
$$X_L = X_C$$
$$\omega_0 L = \frac{1}{\omega_0 C}$$
$$\omega_0^2 = \frac{1}{LC}$$
$$\omega_0 = \frac{1}{\sqrt{LC}}$$
$$2\pi f_0 = \frac{1}{\sqrt{LC}}$$
$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$



$I_z > I_y > I_x$

$R_x > R_y > R_z$

AVERAGE POWER IN A SERIES LCR CIRCUIT

$$P_{avg.} = V_{rms} I_{rms} \cos\phi$$

$$P = VI$$

Let $v = V_0 \sin \omega t$ —— (1)

$i = i_0 \sin(\omega t + \phi)$ —— (2)

$$P = V_0 \sin \omega t \; i_0 \sin(\omega t + \phi)$$
$$= V_0 \sin \omega t \; i_0 [\sin \omega t \cos\phi + \sin\phi \cos \omega t]$$
$$P = V_0 i_0 \left[\sin^2 \omega t \cos\phi + \frac{2 \sin \omega t \cos \omega t}{2} \sin\phi\right]$$

$$= V_0 i_0 \left[\sin^2 \omega t \cos\phi + \frac{\sin 2\omega t}{2} \sin\phi \right]$$

Average power over a complete cycle is equal to

$$= \int_0^T \frac{P\,dt}{T}$$

$$P = \int_0^T \frac{V_0 i_0 \left[\sin^2 \omega t \cos\phi + \frac{\sin 2\omega t}{2} \sin\phi \right] dt}{T}$$

$$= \frac{V_0 i_0}{T} \left[\int_0^T \sin^2 \omega t \cos\phi \, dt + \int_0^T \frac{\sin 2\omega t}{2} \sin\phi \, dt \right]$$

$$= \int_0^T \sin^2 \omega t \cos\phi = \frac{T}{2} \cos\phi$$

$$= \int_0^T \frac{\sin 2\omega t}{2} \sin\phi \, dt = 0$$

$$P_{avg.} = \frac{V_0 i_0}{\cancel{T}} \times \frac{\cancel{T}}{2} \cos\phi$$

$$P_{avg.} = \frac{V_0}{\sqrt{2}} \times \frac{i_0}{\sqrt{2}} \cos\phi$$

$$\boxed{P_{avg.} = V_{rms}\, i_{rms} \cos\phi}$$

$\cos\phi$ is the power factor

$V_{rms} \cdot i_{rms}$ = apparent / virtual power

P = true power

$$\boxed{\cos\phi = \frac{\text{true power}}{\text{virtual power}}}$$

CHOKE COIL -

A choke coil is an electrical device used for controlling current in an AC circuit, without wasting electrical energy in the form of heat.

TRANSFORMER

It is a device which converts high voltage AC into low voltage AC and vice versa. It is based upon the principle of mutual induction.

WORKING AND THEORY

When an AC is passed through the primary coil, the magnetic flux through the iron core changes, which does two things, produces emf in the primary coil and an induced emf is set up in the secondary coil. If we assume that resistance of primary coil is negligible, then the back emf will be equal to the voltage applied to the primary coil.

$$\boxed{V_1 = -N_1 \frac{d\phi}{dt}} \quad \text{and} \quad \boxed{V_2 = -N_2 \frac{d\phi}{dt}}$$

where, N_1 and N_2 are number of turns in the primary and the secondary coil respectively, while V_1 and V_2 are their voltages, respectively.

$$\therefore \frac{\text{Output emf}}{\text{Input emf}} = \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

Energy Losses in a transformer

1. Eddy Current Loss - Eddy current in iron core of transformer facilitate the loss of energy in the form of heat.

2. Flux leakage - Total fluxes linked with primary do not completely pass through the secondary which denotes the loss in the flux or flux leakage.

3. Copper loss - Due to heating, energy loss takes place in copper wires of primary and secondary coils.

4. Hysteresis Loss - The energy loss takes place in magnetising and demagnetising the iron core over every cycle.

5. Humming Loss - The magnetostruction effect leads to set the core in vibration which in turn produced the sound. This loss is referred as humming loss.

TYPES OF TRANSFORMER

1. STEP-UP - ($N_1 > N_2$) It converts low ~~AC~~ alternating voltage into high alternating voltage.
2. STEP-DOWN - ($N_1 < N_2$) It converts high alternating voltage into low alternating voltage.

- For an ideal transformer

Input power = Output power

$$V_1 I_1 = V_2 I_2 \Rightarrow \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

- Transformation ratio (r)

$$r = \frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{I_P}{I_S}$$

- Efficiency

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times 100$$

USES OF TRANSFORMER

1. In the induction furnaces
2. In voltage regulators for TV, computer, etc.
3. For welding purposes.

IMPORTANT QUESTIONS

(1) Obtain the resonant frequency (ω_r) of a series L-C-R circuit with $L = 2.0\,H$, $C = 32\,\mu F$ and $R = 10\,\Omega$. What is the Q-value of this circuit? NCERT

Sol. Given, $L = 2.0\,H$, $C = 32 \times 10^{-6}\,F$ $R = 10\,\Omega$ $\omega_r = ?$ $Q = ?$

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \times 32 \times 10^{-6}}} = \frac{10^3}{8} = 125\ rad/s$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{32 \times 10^{-6}}}$$

$$= \frac{1}{10 \times 4 \times 10^{-3}} = 25$$

---

(2) A 44 mH inductor is connected to 220 V, 50 Hz AC supply. Determine the rms value of the current in the circuit. What is the net power absorbed over a complete cycle? Explain. NCERT

Sol. $L = 44\,mH = 44 \times 10^{-3}\,H$ $V_{rms} = 220\,V$ $\nu = 50\,Hz$

$$X_L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 44 \times 10^{-3} = 13.82\,\Omega$$

The rms value of current in the circuit,

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{220}{13.82} = 15.9\,A$$

Power absorbed, $P = V_{rms} I_{rms} \cos\phi$

For pure inductive circuit, $\phi = 90°$ $P = 0$

Thus, power spent in one half cycle is retrieved in the other half cycle.

---

(3) A coil of inductance 0.5 H and resistance 100 Ω is connected to a 240 V, 50 Hz AC supply.

(i) What is the maximum current in the coil?

(ii) What is the time lag between the voltage maximum and current maximum? NCERT

Sol. Given, $L = 0.5\,H$, $R = 100\,\Omega$

$\nu = 50\,Hz$, $V_{rms} = 240\,V$

(i) $$I_0 = \frac{V_0}{\sqrt{R^2+\omega^2 L^2}} = \frac{\sqrt{2}\times 240}{\sqrt{(100)^2+(100\times\pi\times 0.5)^2}}$$

$= 1.82\,A$ $\qquad [\because \omega = 2\pi\nu = 100\pi]$

(ii) $$\tan\phi = \frac{\omega L}{R} = \frac{2\pi\nu L}{R} = \frac{2\times 3.14\times 50\times 0.5}{100} =$$

$$\phi = \tan^{-1}(3.19\times 10^{-3}5)$$

---

4. A series LCR circuit with $R = 20\,\Omega$, $L = 1.5\,H$ and $C = 35\,\mu F$ is connected to a ~~vehicle~~ variable frequency of 200 V AC supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Sol. When the frequency of the supply equals the natural frequency of the circuit, resonance occurs,

$$Z_r = R = 20\,\Omega$$

$$I_{rms} = \frac{V_{rms}}{Z_r} = \frac{200}{20} = 10\,A$$

$\therefore$ Average power transformed in one cycle

$$P_{av.} = V_{rms} I_{rms} \cos\phi$$

$= 200\times 10\times \cos 0^\circ$ $\qquad [\because \phi = 0^\circ]$

$= 2000\,W = \boxed{2\,KW}$